PMU-Based Monitoring of Power System Dynamics Using Maximum Lyapunov Exponents – TERNA Case Study

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***Abstract*— Power system instability has been a critical and challenging system performance issue for transmission operations and planning engineers. This work utilizes the theory of Maximum Lyapunov Exponent (MLE) along with Synchrophasor Technology to detect potential system instability. First, a new formulation for implementing the MLE technique on power system dynamics is proposed, which enables a fast and accurate detection and characterization of power system dynamic stability performance. Secondly, the effectiveness of the proposed techniques is illustrated on a 537-bus Italian TERNA power system model with simulated and recorded PMU measurement data. The results show that the proposed techniques can detect system instability accurately within a 3-second time window.**

***Index Terms*-- Maximum Lyapunov Exponent (MLE), Phasor Measurement Unit (PMU), Power System Stability, State Calculator (SC), Structure Preserving Model.**

I. Introduction

The availability of high-resolution, synchronized measurements from Phasor Measurement Units (PMUs) presents an opportunity to develop advanced analytical techniques that can evaluate power system stability performance and identify a potential instability condition [1-3]. Such techniques, if implemented in a real-time online environment, can identify abnormal dynamic operating conditions in an early stage, eliminating the associated cost of a potential outage.

Towards this goal, the theory called Maximum Lyapunov Exponent (MLE) has been applied for power system stability assessment [4-8]. A tool called State Calculator (SC) is developed in [5-6] to estimate the unmonitored states from available PMU measurements. The MLE is then calculated for system trajectory within a time window to determine the stability of the trajectory. In [6], a structure preserving model of power systems is applied for MLE technique. This model preserves the topological structure of the system by considering the frequency-dependent characteristics of dynamic loads, achieving a better performance in predicting system stability.

This paper extends the work of [5-6] by making the following two contributions. First, the paper proposes a new formulation for the MLE technique. This new formulation uses a structure preserving model with relative rotor angles, enabling the MLE algorithm to detect the trending information of power system waveforms within a short time window with high accuracy. Secondly, a thorough study on the proposed techniques is made using a 537-bus Italian TERNA power system model (representing an area in Italy), along with simulated and recorded synchrophasor data. A total of 17 simulated fault scenarios, as well as one real event, are studied on the use of system dynamic model, SC estimation accuracy, and MLE prediction accuracy with different window sizes. For all fault scenarios, the MLE technique can provide accurate stability prediction within 3 seconds.

II. Maximum Lyapunov Exponent Technique

A. Maximum Lyapunov Exponent

The Lyapunov exponents are defined to characterize whether a dynamical system is “chaotic” by measuring the exponential divergence or convergence of nearby system trajectories [9]. Consider an *N*-dimensional dynamical system $x(t)=\phi(t,x_0)$ with an initial value x_0. For an infinitesimal

This work is supported by the Electric Power Research Institute (EPRI) and TERNA.

perturbation δx_0 in the initial value, the change at time t can be computed by linear approximation as

$$\delta x(t) = T_x^t \delta x_0 \tag{1}$$

where T_x^t is the Jacobian matrix of $\phi(t, x_0)$ at time t, i.e., $T_x^t = matrix[\partial \phi_i(t, x_0) / \partial x_j]$. The singular values of T_x^t, i.e., $|\Lambda_1|, ..., |\Lambda_N|$, determine the expansion or contraction of the initial perturbation on N orthogonal directions, as shown in Fig. 1. Therefore, the i-th Lyapunov exponent λ_i is defined by the following limit

$$\lambda_i = \lim_{t \to \infty} \frac{1}{t} \ln |\Lambda_i(t, x_0)| \quad i = 1, ..., N . \tag{2}$$

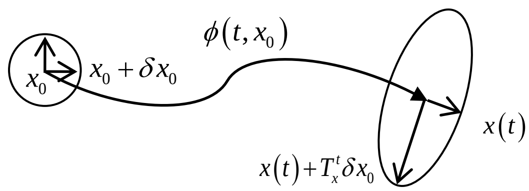


Figure 1. Mapping from an initial perturbation to the change at time t

The existence of limit in (2) is guaranteed by the Oseledec multiplicative ergodic theorem [9]. As time goes to infinity, the largest Lyapunov exponent λ_1, i.e., MLE, dominates exponentially. A negative (positive) λ_1 implies exponential convergence (divergence, respectively) of nearby trajectories. The relation between MLE and the asymptotic behavior of the dynamical system is established in [5]. If the MLE is negative, the waveform of the system is asymptotically stable. Otherwise, it is unstable.

Thus, based on a nonlinear dynamic model of the power system and PMU measurements, the MLE can be used to assess the stability of power systems.

B. Power System Modeling

In this study, the power system is modeled in two ways. Consider a power system with a total of n buses and m generators. The power system can be augmented by m fictitious buses representing the generator internal buses, thus there are m+n buses in the augmented network.

1) Structure preserving model (SP)

The SP model uses the second-order generator model, and the frequency-dependent load model, i.e.,

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ 2H_i \dot{\omega}_i &= P_{Mi} - P_{ei} - D_i \omega_i \end{aligned}, \quad i = 1, ..., m \tag{3}$$

$$P_{dj} = P_{Dj} + D_j \dot{\delta}_j, \quad j = m+1, ..., m+n . \tag{4}$$

The interpretation of the model can be found in [10]. With this model, the original network topology is explicitly represented. The generator voltage is assumed to be constant. The number of states for this model is $2m$+n.

2) Structure preserving model with excitation (SPE)

A more detailed generator model is the fourth-order model with the excitation system. Two extra differential equations modeling the generator voltage in local d-q reference frame are [11]

$$\begin{aligned} \dot{e}'_{qi} &= \frac{1}{T'_{d0i}} \left(E_{fdi} - e'_{qi} - (x_{di} - x'_{di}) i_{di} \right) \\ \dot{e}'_{di} &= \frac{1}{T'_{q0i}} \left(-e'_{di} + (x_{qi} - x'_{qi}) i_{qi} \right) \end{aligned}, \quad i = 1, ..., m . \tag{5}$$

The excitation system is modeled as an integral feedback. That is,

$$\dot{E}_{fdi} = \frac{1}{T_{ei}} \left(E_{refi} - E_{ti} \right), \quad i = 1, ..., m . \tag{6}$$

The SPE model uses (3)-(6), which preserves entire system topology and takes voltage dynamics of generators into account. The number of states for this model is $5m$+n.

C. Implementation

The MLE defined in (1) is based on continuous waveforms with infinite time range. In practice, the Gram-Schmidt Reorthonormalization (GSR) method can be used to obtain the MLE within a finite time window with discrete points. The details of GSR method can be found in [5-6]. Also, the State Calculator in [6] uses the available PMU measurements to estimate the unmonitored system dynamic variables. An index e_x is defined to quantify the SC estimation error. That is

$$e_x = \sqrt{\frac{\sum_{i=1}^{g} \sum_{n=1}^{T_S} \left(x_{i,n}^{SC} - x_{i,n}^{PMU} \right)^2}{g T_S}} \tag{7}$$

in which $x_{i,n}^{SC}$ is the estimated state for i-th trajectory at time step n, $x_{i,n}^{PMU}$ is the simulated PMU measurement for i-th trajectory at time step n, g is the total number of trajectories, T_S is the total number of time steps. A comparison of the accuracy of SC using the index e_x for different models and different numbers and locations of PMUs is presented in Section IV.

Therefore, the MLE algorithm is implemented based on the following procedure. First, given the available PMU measurements, the SC is used to approximate the unobserved states in the system trajectory $x(t)$. Using the synthesized state of the system (observed states by PMUs plus unobserved states calculated by SC), the MLE is calculated within a certain time window T using the GSR method.

Note if the PMU measurements do not provide a sufficient observability level for the system, the estimated dynamic states can be inaccurate [12]. In this study, the time domain simulation is used to generate synthesized PMU data for the MLE calculation.

III. The New MLE Technique Formulation

For implementation in an online environment, the MLE technique needs to satisfy two requirements. First, it should predict the loss of stability with high accuracy. Secondly, the time window it uses for prediction must be short. Therefore, the MLE technique is improved in the following aspects.

First, for the swing equation (3), when the post-fault frequency ω is not 0 p.u., the rotor angle δ will not converge to a constant steady-state value. In this case, the MLE may provide a "false alarm", by predicting a stable system to be unstable. Therefore, to have a post-fault steady state operating point, it is beneficial to define the rotor angle and load bus angle relative to the reference machine, e.g., [11]

$$\delta'_i = \delta_i - \delta_{ref}, \quad \dot{\delta}'_i = \omega_i - \omega_{ref} \qquad (8)$$

By doing so, the dimension of the system dynamic equations is reduced by 1.

Secondly, consider the initial perturbation δx_0 shown on Fig. 1. Theoretically, if MLE is positive, δx_0 at almost all directions will diverge when time goes to infinity. However, since a finite time window is used in practice, it is critical to choose δx_0 at a specific direction that diverges rapidly, such that the instability can be detected at an early stage.

In this study, the initial perturbation direction is chosen as follows: for SP model, the perturbations on ω are chosen as unit value, while the perturbation on δ is 0; For SPE model, the perturbations on e_{qi} and e_{di} are also chosen as unit value.

IV. TERNA Case Study

A case study to evaluate the accuracy and performance of the MLE technique was performed in collaboration with TERNA. A power system model of an area within the Italian TERNA power system, consisting of 537 buses, 18 generators, and 171 loads, was used for this study. 16 PMUs are installed in that area. Time domain simulations of the system were performed using a commercial power systems dynamics tool to provide synthesized PMU data. The simulation results also served as a benchmark for comparison of the SC results.

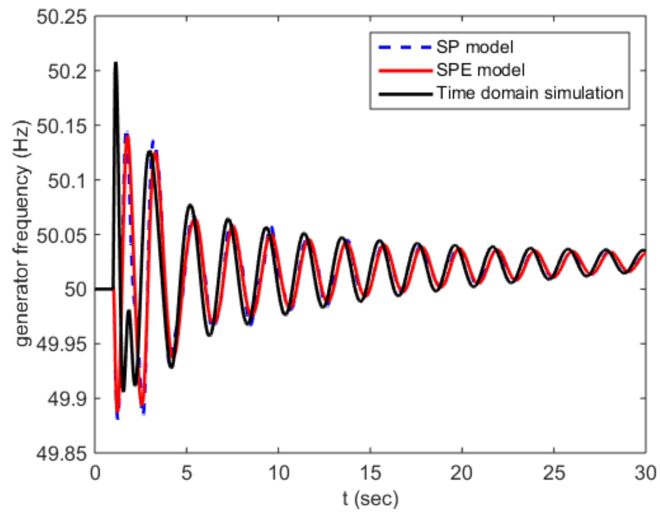
A. SC Accuracy

The accuracy of the SC with 16 PMUs is examined for 17 fault scenarios. Both SP and SPE models were used. The estimation errors defined in (7) are listed in Table I.

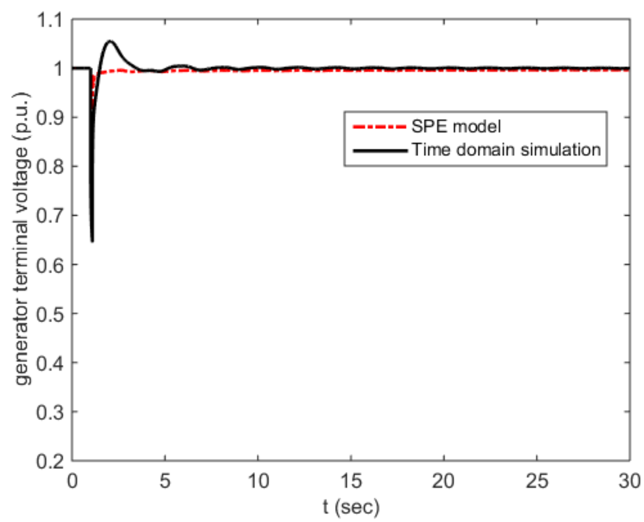
TABLE I. SC Estimation Error

Scenario	**1**	**2**	**3**	**4**	**5**	**6**
SP	0.0513	0.0406	0.0287	0.0046	0.1644	0.1050
SPE	0.0537	0.0583	0.0290	0.0212	0.2467	0.0217
Scenario	**7**	**8**	**9**	**10**	**11**	**12**
SP	0.0343	0.0793	0.0215	0.1032	0.0229	0.1050
SPE	0.0536	0.0966	0.0227	0.0219	0.0220	0.0217
Scenario	**13**	**14**	**15**	**16**	**17**	
SP	0.0370	0.1620	0.0133	0.0212	0.0310	
SPE	0.0332	0.2417	0.0153	0.0222	0.0320	

The comparison results of two representative scenarios are shown in Fig. 2 and Fig. 3. Fig. 2 shows a stable scenario while Fig. 3 an unstable one.

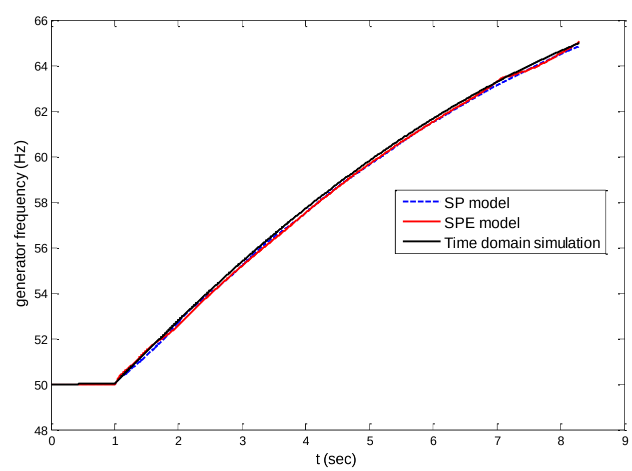


(a) Frequency of generator 289

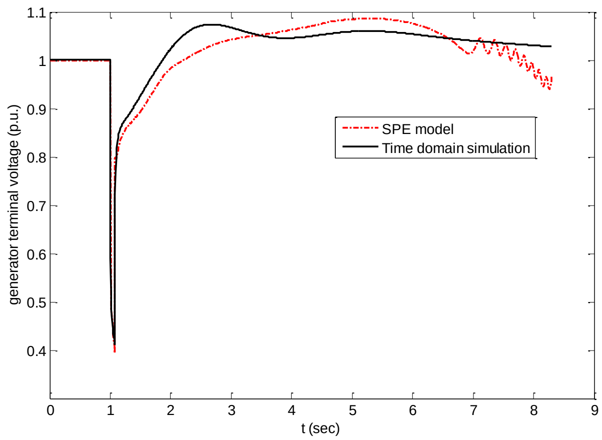


(b) Terminal voltage of generator 289

Figure 2. Comparison of simulation and SC results – stable scenario



(a) Frequency of generator 289



(b) Terminal voltage of generator 289

Figure 3. Comparison of simulation and SC results – unstable scenario

From the diagrams and the table, it is shown that the SC with current PMU placement scheme has small estimation errors for all scenarios and thus can provide accurate estimation results.

B. *Stability Analysis using MLE Technique*

MLE analysis was performed for all the fault scenarios and the results are shown in Table II and Table III. All faults occur at *t*=1 second. *T* is the time window of the MLE analysis. The MLE results are compared with time domain simulation results. In the table, S and U represent stable and unstable scenarios, respectively. The MLE values that are inconsistent with the stability scenarios are made bold.

TABLE II. MLE RESULTS USING SP MODEL

		T=1	*T*=2	*T*=3	*T*=4	*T*=5	*T*=6
1	U	0.1111	0.1454	0.1782	0.2096	0.2396	0.2684
2	S	-0.3139	-1.0109	-1.4021	-1.6619	-1.8606	-2.0204
3	S	**0.0022**	-0.0035	-0.0061	-0.0094	-0.0127	-0.0193
4	S	-0.4919	-1.3381	-1.8617	-2.1703	-2.4146	-2.6282
5	U	0.0635	0.0512	0.0421	0.0382	0.0360	0.0325
6	S	-0.0009	-0.0012	-0.0017	-0.0034	-0.0039	-0.0052
7	S	-0.2025	-0.5837	-0.8108	-1.0088	-1.1571	-1.2848
8	S	-0.0060	-0.0063	-0.0045	-0.0053	-0.0076	-0.0092
9	S	-0.0380	-0.1676	-0.2722	-0.3740	-0.4838	-0.5805
10	S	**0.0072**	-0.0457	-0.1191	-0.2013	-0.2862	-0.3737
11	S	-0.0727	-0.2381	-0.3777	-0.5158	-0.6291	-0.7614
12	S	-0.0009	-0.0012	-0.0017	-0.0034	-0.0039	-0.0052
13	S	**0.3716**	**0.1169**	-0.1689	-0.5118	-0.7677	-0.9782
14	U	0.0560	0.0458	0.0390	0.0360	0.0343	0.0311
15	S	-0.1097	-0.4000	-0.5792	-0.7291	-0.8736	-1.0160
16	S	-0.0005	-0.0548	-0.0915	-0.1361	-0.2002	-0.2583
17	S	-0.0052	-0.0048	-0.0048	-0.0039	-0.0043	-0.0054

TABLE III. MLE RESULTS USING SPE MODEL

		T=1	**T=2**	**T=3**	**T=4**	**T=5**	**T=6**
1	U	0.9129	1.1383	1.3009	1.4282	1.5322	1.6193
2	S	-0.0142	-0.1837	-0.3043	-0.4445	-0.5839	-0.7252
3	S	**0.0259**	**0.0246**	-0.0484	-0.1396	-0.2800	-0.4359
4	S	-0.5393	-1.0039	-1.3741	-1.6314	-1.7951	-1.9280
5	U	0.5085	0.2300	0.1353	0.1120	0.1005	0.0833
6	S	-0.0677	-1.1935	-1.6159	-1.9244	-2.1144	-2.2838
7	S	-0.1156	-0.5527	-0.6994	-0.8063	-0.8653	-0.9356
8	S	-0.0024	-0.0715	-0.1217	-0.1980	-0.2799	-0.3835
9	S	**0.0132**	-0.1254	-0.1718	-0.2829	-0.4371	-0.5937
10	S	**0.0002**	-0.5168	-0.5585	-0.7444	-0.8413	-0.9469
11	S	**0.1086**	-0.3226	-0.4466	-0.6508	-0.7719	-0.8990
12	S	-0.0675	-1.1934	-1.6161	-1.9246	-2.1146	-2.2840
13	S	**0.0085**	-0.1280	-0.1922	-0.3422	-0.4994	-0.6652
14	U	0.2308	0.1256	0.0851	0.0713	0.0659	0.0545
15	S	**0.0012**	-0.2820	-0.1405	-0.2396	-0.2947	-0.3470
16	S	-0.0575	-0.3201	-0.2406	-0.3674	-0.4195	-0.4979
17	S	-0.0333	-0.4175	-0.7052	-0.9950	-1.2689	-1.5299

It is shown that except for scenario 13 for SP model and scenario 3 for SPE model, the MLE can predict system stability with 100% accuracy with a 2-second time window. Even for these two scenarios, a 3-second window is sufficient for accurate prediction. Also, for unstable scenarios 1, 5, and 14, the MLE is able to detect the instability within 1 second. These results validate the effectiveness of the MLE technique for fast detection of potential instability of power systems.

C. *Analysis with Recorded PMU Data*

In addition to the simulation based analysis, a real event in TERNA system was also analyzed. It was a single-phase fault on a line, followed by the opening of the circuit breakers on both sides of the line. Measurements from only one PMU during the event were available. In order to achieve sufficient observability synthesized PMU measurements were created by simulating the event. The comparison of the simulation results and the actual PMU measurement is shown in Fig. 4.

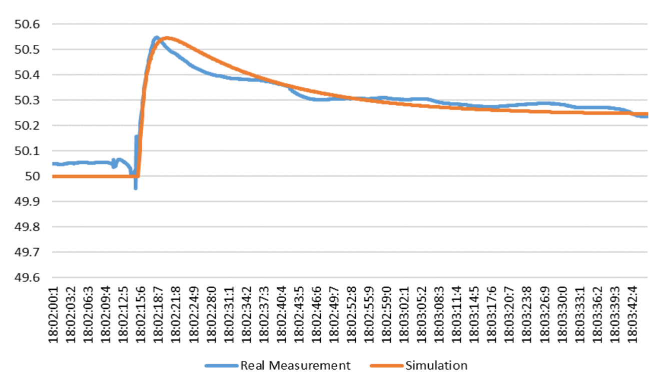
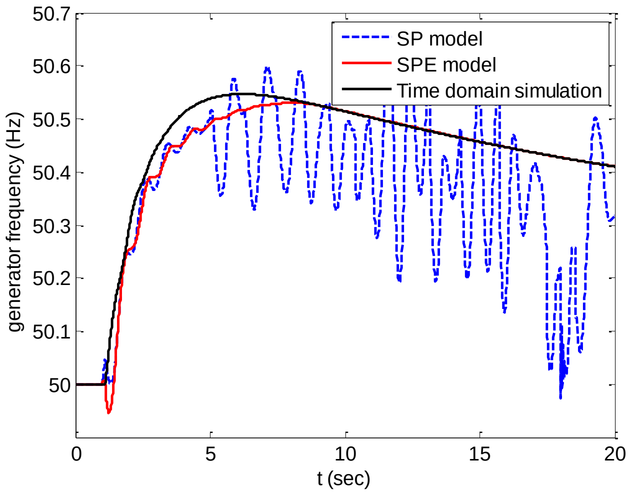
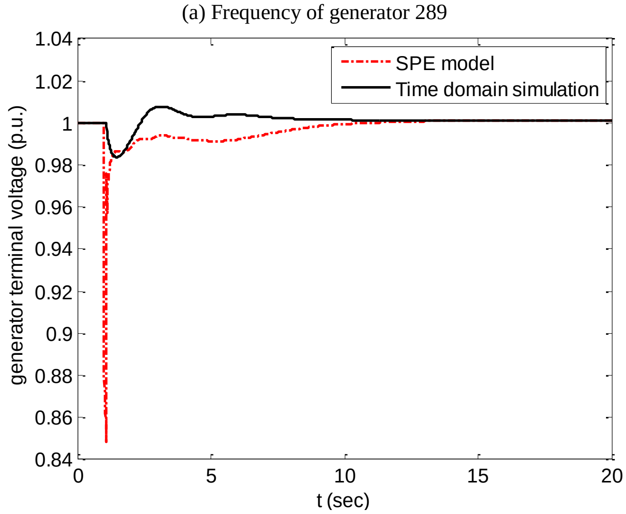


Figure 4. Comparison Between Simulation and Recorded PMU Measurements

The stability of this event was analyzed by SC and MLE, with the recorded PMU measurement along with 15 synthesized PMU measurements. The comparison between time domain simulation results and SC results are shown in Fig. 5.



(a) Frequency of generator 289



(b) Terminal voltage of generator 289

Figure 5. Comparison Between Simulation and Recorded Measurements

It is observed from Fig. 5(a) and Fig. 5(b) that the generator frequency estimation using SPE model has a better estimation result than SP model. The MLE for SP and SPE model with a 1-second time window is 0.1086 and 0.4250, indicating the system is unstable, which is consistent with the simulation results.

V. Conclusion

In this study, an improved technique based on MLE theory is presented for characterization of power system dynamic stability. The technique was evaluated using simulation results and recorded measurements on an area of TERNA. It was shown that the MLE technique with improved formulation can predict system stability performance within 3 seconds with 100% accuracy. The average computational time for SP and SPE model on the study system is 2.23 seconds and 2.83 seconds, with a 20-second simulation length and 5-second MLE window. That shows the MLE technique is promising for an online application.

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