Foreseeing New Control Challenges in Electricity Prosumer Communities

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***Abstract*—This paper is dedicated to electricity prosumer communities, which are groups of people producing, sharing and consuming electricity locally. This paper focuses on building a rigorous mathematical framework in order to formalise sequential decision making problems that may soon be encountered within electricity prosumer communities. After introducing our formalism, we propose a set of optimisation problems reflecting several types of theoretically optimal behaviours for energy exchanges between prosumers. We then discuss the advantages and disadvantages of centralised and decentralised schemes and provide illustrations of decision making strategies, allowing a prosumer community to generate more distributed electricity (compared to commonly applied strategies) by mitigating overvoltages over a low-voltage feeder. We finally investigate how to design distributed control schemes that may contribute reaching (at least partially) the objectives of the community, by resort in to machine learning techniques to extract, from centralised solution(s), decision making patterns to be applied locally. First empirical results show that, even after a post-processing phase so that it satisfies physical constraints, the learning approach still performs better than predetermined strategies targeting safety first, then cost minimisation.**

I. INTRODUCTION

This paper is dedicated to Electricity Prosumer Communities (EPCs), i.e. groups of people producing, sharing and consuming electricity locally. One of the main triggers of the emergence of the concept of energy communities is distributed electricity generation. By distributed electricity generation, we mainly mean PhotoVoltaic (PV) units, small wind turbines and Combined Heat and Power (CHP) that may be installed close to consumers. A cost-drop has been observed over past recent years, especially in the cost of producing PV panels. In addition to this, promises raised by recent advances made in the field of Electric Vehicles (EVs) and batteries may also emphasise in the coming years the metamorphosis of the electricity production, distribution and consumption landscape that is already happening. In addition to electricity production and storage technology improvements, one should also mention the emergence of information technologies facilitating interactions between prosumers [1]. One should also note the existence of projects related to the use of distributed ledgers for managing energy exchanges [2] between microgrids[1].

Our goal is to propose a rigorous mathematical framework for studying energy prosumer communities. We first propose a mathematical framework for modelling the interactions between several prosumers. We then formalise a few optimisation problems targeting several different objectives (e.g., maximising "green" production, taking losses into account, optimising costs and revenues, etc). We address two ways to target these objectives: centralised and distributed control schemes, and we provide examples for each. In the centralised approach, we propose to design a community strategy dedicated to the maximisation of the local renewable energy production by formalising it as an Optimal Power Flow (OPF) problem. Then, in the context where we want to minimise community costs, we propose to design a distributed strategy that may still approach community optimality. To do so, we build upon a recent paper [3] which proposes a centralised optimal strategy named Forward-Backward Sweep OPF (FBS-OPF). We use time series provided by FBS-OPF solutions to build learning sets in the form $(input, output)$, where $input$ contains only local measurements related to one single prosumer, and where $output$ contains an optimal value that was outputted by the FBS-OPF for this input configuration. The learning sets are processed by machine learning techniques in order to build regressors able to compute suggestions for any input configuration.

The remainder of the paper is structured as follows: Section II describes EPCs and the main drivers for their emergence. Section III details our EPC mathematical formalisation, and Section IV specifically focuses on formalising possible objectives for EPCs. Sections V, VI and VII describe control strategies, from centralised to decentralised approaches, to target community objectives, also including numerical results. Section VIII discusses how community control could be extended to unbalanced three-phase networks, and Section IX concludes.

II. THE ELECTRICITY PROSUMER COMMUNITY

A. Definition

An Electricity Prosumer Community (EPC) is a group of electricity consumers and producers who decide to unite in order to reach a specific goal. In many ways, it is similar to a microgrid: it is composed of Distributed Energy Resources (DERs), loads (shiftable or not), possibly storage and EVs. However, it has a key distinction: it is not designed to operate off-the-grid. To reach this specific goal, it uses the flexibility

[1] See for instance the Brooklyn Microgrid project.

of its assets to optimise its behaviour. Furthermore, we will assume that EPCs operate at a local level. It has been suggested to cluster consumers and producers in communities based on their production and consumption profiles regardless of their geographical location. This would be done in order to optimise their relations to energy markets [4]. However, one driver for the community is to increase the hosting capacity of electrical networks and thus, the operational constraints should be taken into account and the impact of the community on the network studied. In this paper, we consider that the energy community is composed of prosumers that are connected to the same low-voltage distribution network and that there is only one point of connection between the community and the power system, which is called the root connection of the community. This means that the power exchanges between the prosumers do not transit through distribution transformers. Examples of such communities include companies in an industrial park, houses in suburban areas, co-ownerships of PV panels installed on rooftops of apartment buildings, etc.

B. Drivers

On the one hand, the drivers for energy communities are clear when an infrastructure is shared between its members, be it DER, storage, CHP or EV charging stations. The installation of such infrastructure is conditioned by the existence itself of a community.

On the other hand, the drivers for a community where all members own their production units or storage are less trivial because they arise from the structure of the energy sector and the behaviour of electrical networks. If we consider communities that exist solely for financial purposes, it is certain that the benefits of the community have to surpass the sum of the benefits of the prosumers if they were isolated, taking into consideration the costs for operating the community. These benefits originate from two sides: network operations and energy markets.

Regarding the first, at the level of a single prosumer, self-consumption by consuming produced power locally allows the reduction of the network losses and the overall use of the networks, and a reduction of the electricity bill. Communities extend the perimeter of self-consumption from one prosumer to several to pool production and flexibility means. This pooling allows for an increased self-consumption of the community, for more optimised power flows, and an increased hosting capacity.

Considering energy markets, by pooling production and flexibility, communities could reach a size where they could offer services to the grid and negotiate their electricity prices.

III. Formalising an Energy Prosumer Community

A. The Prosumers

We consider a set of $N \in \mathbb{N}$ prosumers dynamically interacting with each other over a time horizon $T \in \mathbb{N}$. We first consider a discrete time setting: $t \in \{0, \ldots, T-1\}$. In the following, power related variables are average values over a time window Δ, corresponding to the time interval between two time steps. At every time-step $t \in \{0, \ldots, T-1\}$, each prosumer is characterised by active (resp. reactive[2]) power variables subscripted by P (resp. Q): production variables $P_{P,t}^{(i)}$ and $P_{Q,t}^{(i)}$, (note that $P_{Q,t}^{(i)}$ is positive when producing reactive power and negative when consuming it), a power injected into a storing device $S_t^{(i)}$, the level of charge of the storage device $\lambda_t^{(i)}$, loads (or consumptions) $L_{P,t}^{(i)}$ and $L_{Q,t}^{(i)}$, and powers injected into the distribution network $D_{P,t}^{(i)}$ and $D_{Q,t}^{(i)}$.

We assume that all prosumers may interact with each other. In particular, we denote by $\theta_t^{(i \to j)}$ the (positive) power that is transferred at time t from prosumer (i) to prosumer (j), $(i,j) \in \{1, \ldots, N\}^2$. In the same time, prosumer (j) receives a (positive) power from prosumer (i) denoted by $\theta_t^{(j \leftarrow i)}$. By definition, we have:

$$\forall (i,j) \in \{1, \ldots, N\}^2, \forall t \in \{0, \ldots, T-1\}, \quad \theta_t^{(i \to j)} = \theta_t^{(j \leftarrow i)} \tag{1}$$

with the convention that $\theta_t^{(i \to i)} = 0, \theta_t^{(i \leftarrow i)} = 0 \ \forall i, t$.

At every time step, the active power that is produced by prosumer (i) must satisfy the following relationship:

$$\forall t, i, \ P_{P,t}^{(i)} = L_{P,t}^{(i)} + D_{P,t}^{(i)} + S_{P,t}^{(i)} \tag{2}$$

$$\forall t, i, \ D_{P,t}^{(i)} = \sum_{j=1}^{N} \left(\theta_t^{(i \to j)} - \theta_t^{(i \leftarrow j)} \right) + \delta D_{P,t}^{(i)} \tag{3}$$

where $\delta D_{P,t}^{(i)}$ is the difference between the power injected into the distribution network and the sum of active power exchanges between the members of the community. Note that, in the case where the local production $P_{P,t}^{(i)}$ is not high enough to cover the load $L_{P,t}^{(i)}$, the variable $D_{P,t}^{(i)}$ may take some negative values (depending also on the amount of power that can be taken from the storage device).

The conservation of reactive power at the prosumer's location induces the following:

$$\forall t, i, \ P_{Q,t}^{(i)} = L_{Q,t}^{(i)} + D_{Q,t}^{(i)} \tag{4}$$

In this paper, we focus on energy exchanges among prosumers. For this reason, we choose to neglect electricity losses that are not directly associated with energy exchanges between prosumers.

Prosumers may not always be able to produce electricity at its maximal potential (for instance, PV production may be curtailed when the local storage device is fully recharged, no exchanges between prosumers are possible, and over-voltages are observed on the distribution network because many prosumers are injecting electricity simultaneously: such a situation may appear on sunny days [5], [6]). Thus, for every prosumer, for every time-step, we define the maximal

[2]Considering reactive power is important as it allows greater flexibility in the management of the networks and can allow the community to have more leverage on the network constraints.

production potential which depends on hardware and weather data:

$$\forall t, i \; P_{P,t}^{(i)} \leq P_{P,t}^{(i),\max} \tag{5}$$

$P_t^{(i),\max}$ may be influenced by several parameters, in particular, weather conditions.

The reactive power is limited by the capability curve of the distributed energy resources. It depends on the minimum power factor, the maximum apparent power, and the current active power production.

$$\forall t, i \; \left|P_{Q,t}^{(i)}\right| \leq P_{Q}^{(i),\max}\left(P_{P,t}^{(i)}\right) \tag{6}$$

The injected power into the storage device is capped by a factor that mainly depends on the characteristics of the storage device and its current level of charge:

$$\forall t, i, \; \left|S_t^{(i)}\right| \leq S^{(i),\max}\left(\lambda_t^{(i)}\right) \tag{7}$$

The injected power into the distribution network is also capped, depending on the load and local production, characteristics and level of charge of the battery, and also additional (stochastic) variables, such as weather, that may influence the voltage of the low-voltage feeder (e.g. inability to inject power into the network if the voltage is higher than 1.1 p.u.), thus potentially preventing from power injection:

$$\begin{aligned}\forall t, i, \quad & \left|D_{P,t}^{(i)}\right| \\ \leq D_{P}^{(i),\max} & \left(P_{P,t}^{(i)}, P_{Q,t}^{(i)}, L_{P,t}^{(i)}, L_{Q,t}^{(i)}, S_t^{(i)}, D_{P,t}^{(j\neq i)}, D_{Q,t}^{(j\neq i)}\right)\end{aligned} \tag{8}$$

$$\begin{aligned}\forall t, i, \quad & \left|D_{Q,t}^{(i)}\right| \\ \leq D_{Q}^{(i),\max} & \left(P_{P,t}^{(i)}, P_{Q,t}^{(i)}, L_{P,t}^{(i)}, L_{Q,t}^{(i)}, S_t^{(i)}, D_{P,t}^{(j\neq i)}, D_{Q,t}^{(j\neq i)}\right)\end{aligned} \tag{9}$$

The level of charge of the storage capacity is also bounded:

$$\forall t, i \quad 0 \leq \lambda_t^{(i)} \leq \lambda^{(i),\max} \tag{10}$$

B. The Community

Everything that is not produced by the community has to come from the distribution network through the root of the community. By measuring the active and reactive power transfer at the root, and by comparing the measured powers to those measured at the prosumers' location, we can deduce the losses and the import of reactive power:

$$\forall t \quad \Lambda_{P,t}^{(c)} = D_{P,t}^{(c)} - \sum_{i=1}^{N} D_{P,t}^{(i)} \tag{11}$$

$$\forall t \quad \Lambda_{Q,t}^{(c)} = D_{Q,t}^{(c)} - \sum_{i=1}^{N} D_{Q,t}^{(i)} \tag{12}$$

where $\Lambda_{P,t}^{(c)}$ (resp. $\Lambda_{Q,t}^{(c)}$) denotes the overall losses inside the electrical network of the community (resp. reactive power absorbed by the community network lines), $D_{P,t}^{(c)}$ (resp. $D_{P,t}^{(c)}$) is the active (resp. reactive) power measured at the root of the community.

C. Costs and Revenues

At every time-step, we define a set of price variables, expressed in €/kWh. First, each prosumer (i) may purchase electricity from its retailer at a specific price $Pr_t^{(D\to i)}$. Also, each prosumer (i) may buy electricity from prosumer (j) $(j \neq i)$ at a price $Pr_t^{(j\to i)}$. Each prosumer (i) may also sell electricity to the (distribution) network at a price $Pr_t^{(i\to D)}$, and to other prosumers at a price $Pr_t^{(i\to j)}$. By convention, we assume that all prices considered in the paper are positive. From time t to $t+1$, a prosumer (i) will incur the following cost:

$$\begin{aligned} c_t^{(i)} = \quad & \Delta\Bigg(\max\left(-\delta D_t^{(i)}, 0\right) Pr_t^{(D\to i)} \\ & + \textstyle\sum_{j=1}^{N} \max\left(\theta_t^{(i\leftarrow j)}, 0\right) Pr_t^{(j\to i)}\Bigg)\end{aligned} \tag{13}$$

At the same time, they will also receive the following revenues:

$$\begin{aligned} r_t^{(i)} = \quad & \Delta\Bigg(\max\left(\delta D_t^{(i)}, 0\right) Pr_t^{(i\to D)} \\ & + \textstyle\sum_{j=1}^{N} \max\left(\theta_t^{(j\leftarrow i)}, 0\right) Pr_t^{(j\to i)}\Bigg)\end{aligned} \tag{14}$$

D. Community Dynamics

The variables dynamically evolve over time, also suffering some stochasticity. We define a state vector Ξ_t as being the collection of all (measurable) variables related with the physical characteristics of the system, and a price vector Φ_t gathering all prices : $\forall t \in \{0, \ldots, T-1\}$,

$$\Xi_t = \begin{pmatrix} P_{P,t}^{(1)} & P_{Q,t}^{(1)} \\ P_{P,t}^{(1),\max} & P_{Q,t}^{(1),\max} \\ \vdots & \vdots \\ P_{P,t}^{(N)} & P_{Q,t}^{(N)} \\ P_{P,t}^{(N),\max} & P_{Q,t}^{(N),\max} \\ S_t^{(1)} & \lambda_t^{(1)} \\ \vdots & \vdots \\ S_t^{(N)} & \lambda_t^{(N)} \\ L_{P,t}^{(1)} & L_{Q,t}^{(1)} \\ \vdots & \vdots \\ L_{P,t}^{(N)} & L_{Q,t}^{(N)} \\ D_{P,t}^{(1)} & D_{Q,t}^{(1)} \\ \vdots & \vdots \\ D_{P,t}^{(N)} & D_{Q,t}^{(N)} \end{pmatrix}, \quad \Phi_t = \begin{pmatrix} Pr_t^{(D\to 1)} \\ Pr_t^{(1\to D)} \\ \vdots \\ Pr_t^{(D\to N)} \\ Pr_t^{(N\to D)} \\ Pr_t^{(1\to 2)} \\ Pr_t^{(2\to 1)} \\ \vdots \\ Pr_t^{(1\to N)} \\ Pr_t^{(N\to 1)} \\ \vdots \\ Pr_t^{(N-1\to N)} \\ Pr_t^{(N\to N-1)} \end{pmatrix} \tag{15}$$

We also define two series of matrices. The first series $\Theta_t^{\rightarrow}$ is related to energy exchanges between prosumers according the the producer's point of view, whereas the second series $\Theta_t^{\leftarrow}$

is written according to the receiver's (or consumer's) point of view:

$$\Theta_t^{\rightarrow} = \left(\theta_t^{(i \rightarrow j)}\right)_{i,j}, \qquad \Theta_t^{\leftarrow} = \left(\theta_t^{(i \leftarrow j)}\right)_{i,j} \tag{16}$$

Since it may not be easy to assess whether the system defined through the previously described state vectors is Markovian or not, we have : $\forall t \in \{0, \ldots, T-1\}$,

$$\Xi_{t+1} = F(\Xi_t, \Phi_t, \Theta_t^{\rightarrow}, \Theta_t^{\leftarrow} \ldots, \Xi_0, \Phi_0, \Theta_0^{\rightarrow}, \Theta_0^{\leftarrow}, \omega_t) \tag{17}$$

where $\omega_t \in \Omega$ is an exogenous random variable drawn according to an exogenous, time-dependent probability distribution $\omega_t \sim P_t(\cdot)$.

IV. New Control Challenges

In this paper, we focus on the formalisation of decision making problems within a community of energy prosumers. Many control algorithms have already been proposed in literature, however, without specifically approaching it from a community angle (see for example [7]–[9]). By decision making, we mean that at every time step, prosumers have the opportunity to make several decisions: (i) Adapting their level of production and/or consumption, (ii) buying/selling to other prosumers and (iii) buying /selling to the retailer. In the following, we detail a few optimisation criteria that may be considered when optimising a community of prosumers.

A. Maximising the distributed production

As briefly discussed previously, it may occur that decentralised production may by curtailed, mainly because load, storage and the distribution network may not be able to host it on some sunny days. It may make sense to investigate control strategies dedicated to maximising decentralised production. More formally, one may seek to optimise, over the time horizon T, the production of decentralised electricity:

$$\max_{\substack{P_{P,t}^{(i)}, P_{Q,t}^{(i)}, L_{P,t}^{(i)}, L_{Q,t}^{(i)}, S_t^{(i)}, \Theta_t^{\rightarrow}, \Theta_t^{\leftarrow} \\ t \in \{0,\ldots,T-1\} \\ i \in \{1,\ldots,N\}}} \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{i=1}^{N} P_{P,t}^{(i)}\right] \tag{18}$$

while satisfying all constraints and time coupling between time steps.

Another optimisation criterion that may be of interest is to optimise distributed production while also limiting losses due to energy exchanges:

$$\max_{\substack{P_{P,t}^{(i)}, P_{Q,t}^{(i)}, L_{P,t}^{(i)}, L_{Q,t}^{(i)}, S_t^{(i)}, \\ \Theta_t^{\rightarrow}, \Theta_t^{\leftarrow} \\ t \in \{0,\ldots,T-1\} \\ i \in \{1,\ldots,N\}}} \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{i=1}^{N} P_{P,t}^{(i)} - \Lambda_{P,t}^{(c)}\right] \tag{19}$$

while satisfying all constraints and time coupling between time-steps.

B. Optimising overall costs and revenues

Costs and revenues may be globally optimised by optimising the overall costs and revenues of the prosumer community:

$$\max_{\substack{P_{P,t}^{(i)}, P_{Q,t}^{(i)}, L_{P,t}^{(i)}, L_{Q,t}^{(i)}, S_t^{(i)}, \\ \Theta_t^{\rightarrow}, \Theta_t^{\leftarrow} \\ t \in \{0,\ldots,T-1\} \\ i \in \{1,\ldots,N\}}} \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{i=1}^{N} r_t^{(i)} - c_t^{(i)}\right] \tag{20}$$

while satisfying all constraints and coupling between time-steps.

V. Control strategies

To achieve the objectives formalized in the previous section, two classes of control strategies compete with each other: a centralised one and a distributed one. All control strategies require controllable inverters, batteries, charging stations for active and reactive power. Controllable loads can also be considered. More specifically, in a centralised scheme, the modulation orders are computed by a centralised entity responsible for gathering the data, computing the orders and sending them to the prosumers. In a distributed scheme, all actors compute their own actions based on local objectives and measurements. The choice for a control strategy depends on several assumptions regarding the available information on the network (a detailed electrical model, estimation of the distance between the prosumer and the distribution transformer, etc.), the presence of communication (GPRS, PLC, Broadband, etc.), the presence of storage or, a central controller.

VI. Centralised schemes

A. Technical challenges for building the centralised scheme

A centralised control scheme comprises three different parts. The first part is all the elements on which it relies for acquiring information about the system it controls. The second part is the "brain" of the scheme, something that is usually called the controller in the control literature. It computes, from the (history of) information, control actions. The third and last part is the infrastructure used for sending and applying its control actions. In the next subsections, we discuss the main elements of infrastructure that need to be put in place to build a centralised control scheme.

1) Information gathering: This part is typically composed of sensors used for measuring physical values, and of a communication infrastructure for sending them to the controller.

A centralised control scheme needs a full knowledge of the system. Therefore, the infrastructure needs to have: (i) Sensors able to measure the power consumed by the loads, the current state of charge of the batteries, estimation of the maximum production of DERs, etc. and (ii) communication channels able to transfer these measurements from the houses to the centralised controller. As communication channels, different technologies exist. For example, internet connections or General Packet Radio Service (GPRS) connections can be used. We could also think about using Power Line Communication (PLC) that carries data on the AC line. A mix of several communication technologies could also be used. For example, the data from the houses could be transmitted using a PLC-based technology to the nearest substation, from which GPRS technology would be used for transferring them to the centralised controller.

2) The centralised controller for processing the information: The second part of the infrastructure is related to the machinery needed for storing the information gathered about the system, processing this information, and computing the control actions based on measurements.

3) From computational results to applied actions: Once the actions have been computed by the centralised controller, they need to be applied to the system. This implies having a communication channel between the centralised controller and the inverters. This also implies having inverters which are able to modify, upon request, the amount of active and reactive power injected into the network.

B. Local Energy Markets

As a way to target the goals of an EPC, it has been suggested creating local energy markets to generate *incentives that boost investment in DER while at the same time creating enticements for containing and balancing the renewable energy produced* [10]. In this paper, the authors propose a combined market model for energy, flexibility and services at the neighbourhood level. The market is managed by an SESP (Smart Energy Service Provider) which can operate as a broker when local trades are peer-to-peer, as a retailer for over-the-counter sales with bilateral contracts, or as a market maker when a call auction is necessary.

C. Optimal Power Flow

Another scheme to solve the control challenges in a centralised fashion is to use Optimal Power Flow techniques, where the objectives and the constraints are the ones in Section III. In addition to those constraints, power flow constraints are added to link the powers injected at the node of the network to the voltages. Several methods exist to solve such problems, such as in [11]. A method of particular interest is the one developed by Fortenbacher et al. [3] where they adapt the Forward-Backward Sweep algorithm to OPF by linearising the power flow equations, given common assumptions that can be made in low-voltage distribution networks such as high R/X ratio, small angles deviation, etc.

D. A first illustration: optimising PV injection over the network without storage

In this section, we assume that the low-voltage feeder gathers N houses, each of them being provided with a photovoltaic installation. We provide an illustration of the network in Figure 1. Presuming a deterministic setting, the following experiments show how to control the active power injected into the distribution network by each inverter for each time step in order to maximise the overall injected power while avoiding over-voltages:

$$\forall t \; \left(P_{P,t}^{(1),*}, \ldots, P_{P,t}^{(N),*}\right) \in \underset{P_{P,t}^{(1)},\ldots,P_{P,t}^{(N)}}{\arg\max} \; \sum_{i=1}^{N} P_{P,t}^{(i)} \tag{21}$$

subject to operational constraints.

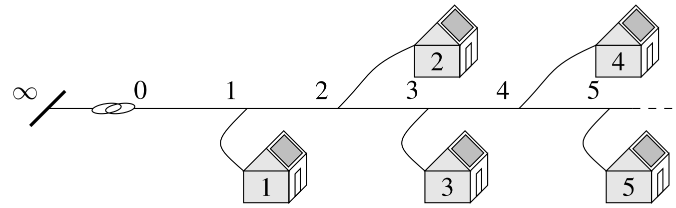

Fig. 1. Graphic representation of the test network.

In the following, we assume that:

- The electrical distances between two neighbouring houses are the same and all electrical cables have the same electric properties,
- The line resistance is $0.24\ \Omega\ \mathrm{km}$,
- The line reactance is $0.1\ \Omega\ \mathrm{km}$,
- The distance between houses $50\ \mathrm{m}$,
- The nominal voltage of the network is 400 V,
- The value of the impedance of the Thévenin equivalent Y_{Th} is equal to $0.0059 + j0.0094\ \Omega$,
- The value of the Thévenin voltage is equal to 420 V.

As a consequence, for having a fully defined energy-based prosumer community; we just need to define the four following quantities:

- The number of houses is set to $N = 18$,
- The impedance between two neighbouring houses,
- The load profile, for every house and every time-step.

In Figure 2, we provide a graph of the evolution of the PV energy production for all the houses of the feeder. In Figure 3,

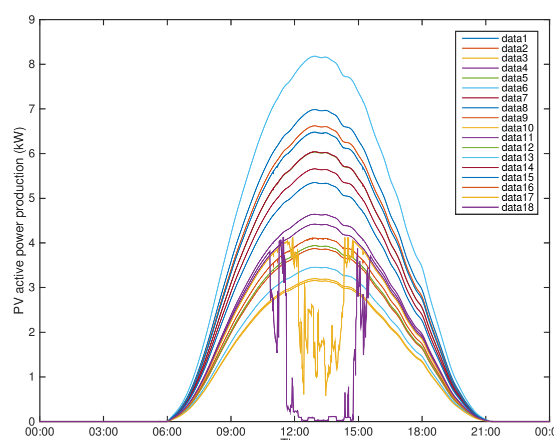

Fig. 2. Illustration of the PV production of the 3 last houses of the low-voltage feeder over 24 hours.

we provide a graph of the evolution of the voltage for all buses of the feeder. One can observe that the production of

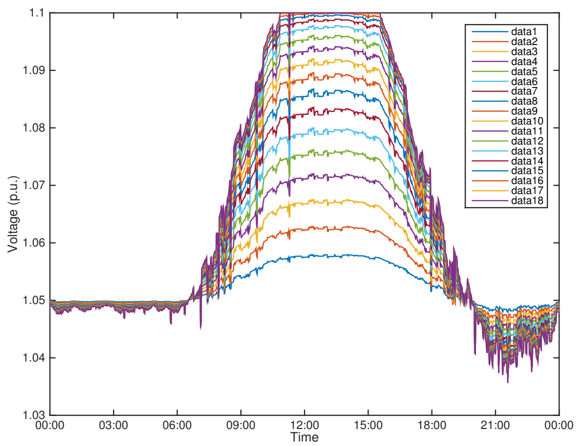

Fig. 3. Illustration of the voltage potential over 24 hours.

houses located at the end of the feeder (i.e., far from the transformer) is modulated in order to avoid over-voltages. Even if the community still suffers partial curtailment, it has to be compared with the complete disconnection of PV units when overvoltages are observed. In the centralised community strategy, the total curtailment was 21.38 kWh, whereas the complete disconnection of inverters observing an overvoltage at their bus would lead to a curtailment of 31.63 kWh.

VII. Distributed Schemes

In the previous section, we have proposed a centralised control scheme to suppress overvoltages in the community. As specified in Section VI-A, one of the main shortcomings of centralised controllers is their cost of implementation and maintenance. They indeed require building and maintaining a costly communication infrastructure between the houses and the centralised controllers. They also require a detailed model of the low-voltage network that may be expensive to obtain. Therefore, it would be interesting to design other types of control schemes that would be much cheaper. Ideally, these schemes should not rely on an expensive communication architecture and should be able to work even without knowing a detailed model of the low-voltage network.

In this section, we investigate how to design distributed control schemes that may contribute to reaching (at least partially) the objectives of the community. Our strategy is to resort to machine learning techniques that may extract, from centralised solution(s), decision making patterns to be applied locally, i.e. by only measuring features about the (local) prosumer. Our machine learning approach is an imitative learning-type approach where we learn four different regressors from data. These four regressors are dedicated to learning the optimal levels of active power production, reactive power production, power injected into the storage device and power drawn from the storage device to be applied by prosumer (i).

A. Generating data for solving machine learning tasks

First, for each prosumer $i \in \{1, \dots, N\}$, we generate a set of load profiles $L_{P,t}^{(i)}, t \in \{1, \dots, T\}$ and maximal production potentials $P_{P,t}^{(i),max}, t \in \{1, \dots, T\}$, associated with a time series of price vectors $\phi_t, t \in \{1, \dots, T\}$. We consider the optimisation criterion described in Equation 20. This power flow problem can be solved using, for instance, the FBS-OPF algorithm proposed by [3]. Solving one such problem outputs a time series of data, corresponding to the evolution of all the indicators over the time horizon:

$$\left[\Xi_0^*, \dots, \Xi_{T-1}^*\right] \tag{22}$$

From this time series of data, one can extract a series of local data, i.e. relative to one single prosumer (i):

$$\left[\Xi_0^{(i),*}, \dots, \Xi_{T-1}^{(i),*}\right] \tag{23}$$

where $\forall t \in \{0, \dots, T-1\}$, $\forall i \in \{1, \dots, N\}$,

$$\Xi_t^{(i),*} = \begin{pmatrix} P_{P,t}^{(i),*} & P_{Q,t}^{(i),*} \\ P_{P,t}^{(i),\max} & P_{Q,t}^{(i),\max} \\ S_t^{(i),*} & \lambda_t^{(i),*} \\ L_{P,t}^{(i)} & L_{Q,t}^{(i)} \\ D_{P,t}^{(i),*} & D_{Q,t}^{(i),*} \end{pmatrix}, \tag{24}$$

From these extractions, we generate the following learning sets:

- For generating a learning set dedicated to learning how to optimize the level of active power production, we process the variables $\Xi_t^{(i),*}$ into the following set of (input, output) pairs:

$$\mathcal{L}^P = \left\{ \left(in_P^{i,t}, out_P^{i,t} \right) \right\}_{i=1,t=0}^{i=N,t=T-1} \tag{25}$$

  where, $\forall t \in \{0, \dots, T-1\}$, $\forall i \in \{1, \dots, N\}$,

$$in_P^{i,t} = \left(i,\ |\overline{\mathbf{v}}_t^{(i)}|,\ arg(\overline{\mathbf{v}}_t^{(i),*}),\ \phi_t,\ \lambda_t^{(i),*},\ L_{P,t}^{(i)},\ P_{P,t}^{(i),max} \right) \tag{26}$$

$$out_P^{i,t} = P_{P,t}^{(i),*} \tag{27}$$

  where:
  - i : id number of the bus
  - $|\overline{\mathbf{v}}_t^{(i)}|$: magnitude of the voltage at bus i at time step t
  - $arg(\overline{\mathbf{v}}_t^{(i)})$: phase of the voltage at bus i at time step t
  - ϕ_t : electricity price at time step t, considered as being unique in the whole feeder
  - $\lambda_t^{(i)}$: level of charge of the storage of bus i at time step t
  - $L_{P,t}^{(i)}$: load consumption at bus i at time step t
  - $P_{P,t}^{(i),max}$: maximal production potential at bus i at time step t
- For generating a learning set dedicated to learning how to optimize the level of reactive power production, we

process the whole variables $\Xi_t^{(i),*}$ into the following set of (input, output) pairs:

$$\mathcal{L}^Q = \left\{ \left(in_Q^{i,t}, out_Q^{i,t} \right) \right\}_{i=1,t=0}^{i=N,t=T-1} \tag{28}$$

where, $\forall t \in \{0,\ldots,T-1\}$, $\forall i \in \{1,\ldots,N\}$,

$$\begin{aligned} in_Q^{i,t} &= \; = \; in_P^{i,t} \\ out_Q^{i,t} &= \; = \; P_{Q,t}^{(i),*} \end{aligned}$$

- For generating a learning set dedicated to learning how to optimize the level of power injected into the battery, we process the whole variables $\Xi_t^{(i),*}$ into the following set of (input, output) pairs:

$$\mathcal{L}^C = \left\{ \left(in_C^{i,t}, out_C^{i,t} \right) \right\}_{i=1,t=0}^{i=N,t=T-1} \tag{29}$$

where, $\forall t \in \{0,\ldots,T-1\}$, $\forall i \in \{1,\ldots,N\}$,

$$\begin{aligned} in_C^{i,t} &= in_P^{i,t} \\ out_C^{i,t} &= \max\left(S_t^{(i),*}, 0 \right) \end{aligned}$$

- For generating a learning set dedicated to learning how to optimize the level of power injected into the battery, we process the whole variables $\Xi_t^{(i),*}$ into the following set of (input, output) pairs:

$$\mathcal{L}^D = \left\{ \left(in_D^{i,t}, out_D^{i,t} \right) \right\}_{i=1,t=0}^{i=N,t=T-1} \tag{30}$$

where, $\forall t \in \{0,\ldots,T-1\}$, $\forall i \in \{1,\ldots,N\}$,

$$\begin{aligned} in_D^{i,t} &= in_P^{i,t} \\ out_D^{i,t} &= \max\left(-S_t^{(i),*}, 0 \right) \end{aligned}$$

The machine learning task is performed using Extremely Randomized Trees [12] using the Scikit-learn library [13].

B. Post-processing the predictions

When the regressors learned from data are used to set the value of a decision variable inside the community, their output needs to be post-processed, otherwise it could create a violation of physical constraints (e.g. the predicted value of power drawn from the storage is greater than the power that the storage can offer). In that case, the value is corrected and set equal to the limit that it crossed (e.g. the power drawn from the storage becomes equal to the maximum power that the storage can offer).

C. Applying the learned strategies in different load, solar production and prices configurations

A new set of load profile time series $L_{P,t}^{(i)}, t \in \{1,\ldots,T\}$ and maximal production potentials time series $P_{P,t}^{(i),max}, t \in \{1,\ldots,T\}$, associated with a new time series of price vectors $\phi_t, t \in \{1,\ldots,T\}$ are generated for each prosumer $i \in \{1,\ldots,N\}$. Starting from the initial time step, at every t, the required inputs are passed to the regressors for each prosumer and the outputs (after a post-processing step) are used to set the value of their actions. The power flow problem is solved every time to check the voltages, the net power exchanged with the main grid and respect of the physical constraints.

D. Empirical illustration

In this section, we compare the performance of the learned strategies in a deterministic setting with two other strategies: (i) the centralised optimised strategy as defined in [3], and (ii) another decentralised strategy relying on a predetermined, thresholds-based, decision rule. This second decentralised strategy is designed so that it ensures the safety of the system, and then, tries to restrain the overall costs of the community. The first point of this second decentralised algorithm is, thus, to check if there is a risk of overvoltages or undervoltages at the bus and, in this case, to orient the actions of that prosumer to avoid it (fully charging/discharging the storage and maximising/minimising the power production). In the case where the safety of the grid seems ensured, the decisions are imposed based on the price of the electricity at that time step (when it is above/under a predetermined price, impose a predetermined prosumer's action). It is certainly simplistic, but it has the merit of providing a comparison base. Details about this decision rule can be found in Appendix.

As a comparison metric, we consider the overall costs that the community incurs (in the same overall environment, i.e. same loads, solar production, PV and batteries sizes, prices) exchanging power with the main grid during an entire year ($T = 8760$, one time-step per hour during one year). The comparison is made in an environment where loads, solar production and prices are not the same as the one from which the learned strategies were built. As expected, the centralised model is able to achieve the lowest costs, equal to 641.70 €. If we adopt the predicted actions made by the learned regressors, the community meets a total cost of 1549.70 €, a result that seems expensive when compared to the centralised model one, but it becomes remarkable when we consider that the "rule of thumb" algorithm produces an expense equal to 3276 €.

VIII. One step further: taking into account the three phases

The mathematical formalisation presented in this paper considers a balanced operation of the network. Indeed, the power exchanges between the prosumers do not take into consideration the phase to which they are connected. It considers only one value for active and reactive power per dwelling. However, low-voltage distribution networks are intrinsically unbalanced because even if a prosumer has a three-phase connection to the grid, house appliances are mainly single phase. Our concern is the relevance of exchanging powers between members of the electricity community, that are not connected to the same phase. Physically, current from the DER would flow to the distribution transformer and out of the community while current to supply the load would flow from the distribution transformer. While this may reduce the losses to some extent because power does not flow from the transmission network, controlling the community in this fashion could further unbalance the network, and result in the violation of voltage constraints and a reduction of the hosting capacity of the network. One solution would be to divide the community into three groups: one per phase and prosumers that have a three-phase connection would belong to the three groups. This would allow the application of the same formalism and would ensure that exchange of powers takes place solely between prosumers that are connected to the same phase.

IX. Conclusion

This paper proposes a mathematical framework for modelling energy exchanges between prosumers. In particular, it allows formalising a family of optimisation problems, depending on the target objective (maximising green production, minimising losses, optimising revenues), and also on the structure of the control strategies (centralised or decentralised). In particular, we have proposed a machine learning approach whose objective is to mimic, at an individual level (i.e., using local measurements only), a behaviour that is optimal at the community level. First empirical results show that, even after a post-processing phase so that it satisfies physical constraints, the learning approach still performs better than predetermined strategies targeting safety first, then cost minimisation.

Future work includes designing decentralized strategies relying on other data-based approaches, in particular, we think Reinforcement Learning (RL) [14] could be a powerful paradigm to learn decentralised strategies, mainly because RL agents have the characteristic to self-improve over time with new data acquisition.

Appendix

We define $\eta_c^{(i)}$ as the charge efficiency and $\eta_d^{(i)}$ as the discharge efficiency of the storage of prosumer $i \in \{1, \dots, N\}$. We set arbitrary values for ϕ^- and ϕ^+, that are thresholds, respectively, for what can be considered a low price and an high price for electricity. At every time-step $t \in \{0, \dots, T-1\}$ and for each prosumer $i \in \{1, \dots, N\}$ the "rule of thumb" algorithm used in Section VII is structured as follows:

if $|\overline{\mathbf{v}}_t^{(i)}| \leqslant 0.91pu$
  $P_{P,t}^{(i)} = P_{P,t}^{(i),max}$
  $P_{Q,t}^{(i)} = P_{Q,t}^{(i),max}$
  $S_t^{(i)} = -\lambda_t^{(i)}\ \eta_d^{(i)}$
else if $|\overline{\mathbf{v}}_t^{(i)}| \geqslant 1.09pu$
  $P_{P,t}^{(i)} = 0$
  $P_{Q,t}^{(i)} = -P_{Q,t}^{(i),max}$
  $S_t^{(i)} = \frac{\lambda_t^{(i),max} - \lambda_t^{(i)}}{\eta_c^{(i)}}$
else
  $P_{P,t}^{(i)} = P_{P,t}^{(i),max}$
  $P_{Q,t}^{(i)} = 0$
  if $\phi_t \geqslant \phi^+$
    if $P_{P,t}^{(i)} \geqslant L_{P,t}^{(i)}$
      if $\lambda_t^{(i)} \geqslant 0.3\ \lambda_t^{(i),max}$
        $S_t^{(i)} = -\left(\lambda_t^{(i)} - 0.3\ \lambda_t^{(i),max}\right)\eta_d^{(i)}$
      else
        $S_t^{(i)} = 0$
    else
      $S_t^{(i)} = -\lambda_t^{(i)}\ \eta_d^{(i)}$
  else if $\phi_t \leqslant \phi^-$
    if $P_{P,t}^{(i)} \geqslant L_{P,t}^{(i)}$
      if $P_{P,t}^{(i)} - L_{P,t}^{(i)} \leqslant \left(\lambda_t^{(i),max} - \lambda_t^{(i)}\right)\eta_c^{(i)}$
        $S_t^{(i)} = \frac{P_{P,t}^{(i)} - L_{P,t}^{(i)}}{\eta_c^{(i)}}$
      else
        $S_t^{(i)} = \frac{\lambda_t^{(i),max} - \lambda_t^{(i)}}{\eta_c^{(i)}}$
    else
      if $\lambda_t^{(i)} \geqslant 0.3\ \lambda_t^{(i),max}$
        $S_t^{(i)} = -\left(\lambda_t^{(i)} - 0.3\ \lambda_t^{(i),max}\right)\eta_d^{(i)}$
      else
        $S_t^{(i)} = \frac{0.3\ \lambda_t^{(i),max} - \lambda_t^{(i)}}{\eta_c^{(i)}}$
if $S_t^{(i)} > S_t^{(i),max}$
  $S_t^{(i)} = S_t^{(i),max}$
if $S_t^{(i)} < -S_t^{(i),max}$
  $S_t^{(i)} = -S_t^{(i),max}$

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