Robust optimization taking into account forecasting errors and corrective actions

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***Abstract*—This article deals with day-ahead security management taking into account uncertainties on the next day generation and load. Mixed-integer bilevel optimization allows to test the efficiency of corrective actions after the tripping of lines with respect to flow limits on the French EHV/HV network. A first crucial issue faced by operation planning engineers is to examine critical situations which may include unexpected demand and the tripping of generators which was not possible in the formulation proposed in our previous paper [4]. The second improvement is to formulate a new problem optimizing topological actions in order to find an unique solution robust against all uncertainties. For achieving this goal, we propose a semi-infinite program for solving this challenging optimization problem.**

***Index Terms*—operation planning, intraday operation, security management under uncertainty, transmission system operator, worst case analysis, mathematical programming**

I. OUTLINE

DAY-AHEAD operational planning as well as intraday operation of power systems is affected by an increasing amount of uncertainty due to the coupling of wind power intermittency, cross-border interchanges, market clearing, and load evolution. It may be formalized in the form of a three stage sequential decision making problem, where the successive stages correspond to different decision variables, namely

- $u_s \in U_s$, denotes the *strategic* planning decisions that must be taken the day ahead (or possibly postponed) such as reserve purchase and/or maintenance rescheduling;
- $u_p \in U_p$, denotes the *preventive* control actions that may be taken in normal operation to ensure security, such as topology switching, generation rescheduling, Var-control;
- $u_c \in U_c$, represents *corrective* controls that may be applied in emergency mode, after the occurrence of a contingency (e.g. fast generation control, load shedding).

In this paper we consider the problem of checking for given day ahead decisions and preventive control actions (say $\hat{u}_s$ and $\hat{u}_p$) whether or not the system security will be manageable for the next day. In other words, we ask whether or not for all scenarios $s \in S$, the possible combinations of corrective controls will be sufficient to maintain the system in acceptable conditions during the next day.

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II. FORECAST ERROR CHARACTERIZATION ON THE FRENCH EHV/HV NETWORK

Uncertainties affect generators and loads named elementary injections. The plausible domain, computed on the historical data through linear constraints, contains elementary injections that are likely enough to be reached. More precisely, an injection vector s, consisting of active powers for generators and loads, is in the plausible domain ($s \in S$) if there exists r such that :

$$s = f + Ar$$
$$r_{min} \leq r \leq r_{max}$$

where :

- f is the vector of forecasted injections (e.g. day ahead)
- r is the vector of reduced errors
- A is the reconstruction matrix (N,m) obtained through a principal component analysis
- r_{min}, r_{max} are vectors defining the confidence intervals

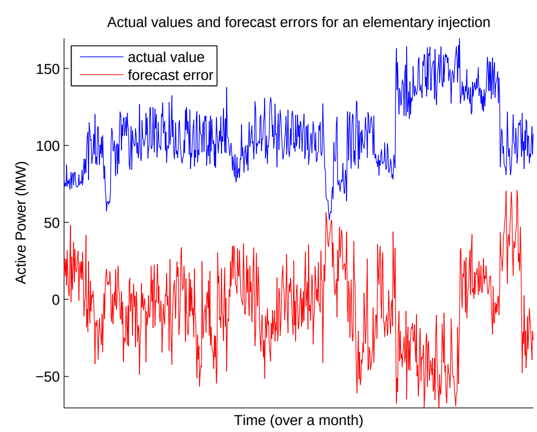

Fig. 1. Sample of load connected in a 225kV substation

Figure 1 highlights the need for a plausible domain which captures the correlations between uncertainties. Indeed, very pessimistic and unrealistic scenarios are generated if individual variations of loads are considered as independent. By comparison, figure 2 below depicts the results of optimizations $\max / \min \ (f + A \ r)_k \ s.t. \ r_{min} \leq r \leq r_{max}$ for all elementary injections k, the less pronounced variations are explained by the fact that $m = 143 \ << \ N = 5709$.

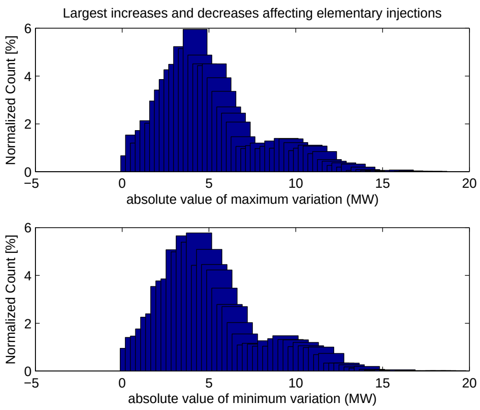

Fig. 2. Absolute value of max and min variations deduced from the plausible domain

III. Line overload problems after contingency when total power balance changes

To determine the most severe violation of flow limits under uncertainties, we adopt the DC approximation and express corrective controls through linear relations. As in Ref. [4], the phase shifting transformers (PST) form the corrective actions, however the innovation here is the introduction of the secondary load-frequency control in balancing which allows us to simulate unexpected demand and the tripping of generator units :

$$\max_{\substack{t\in\{0,1\},\alpha\in[-1,+1]\\ \theta,s\in S,p\in\{0,1\},d\in\{0,1\},e\in\{0,1\},u_c,\lambda}} \lambda \tag{1}$$

$$\textbf{s.t.}\begin{cases}\underline{u}_p \leq \hat{u}_p + \delta \leq \bar{u}_p\\ F(\theta,\hat{u}_p+\delta,s)=0 \quad \sum s = \sum_g \delta_g \quad (balancing)\\ -(L_2+L_1)(1-p)+L_1 d \leq M_\theta\ \theta \leq L_1+(L_2-L_1)d\\ -(L_2+L_1)p+L_1 d \leq -M_\theta\ \theta \leq L_1+(L_2-L_1)d\\ \forall\ g\in\mathcal{G}\\ \quad B\ (t-1)\leq \delta_g \leq B\ t\\ \quad |\delta_g - f_g\ \alpha| \leq B\ (M_g+m_g)\\ \quad |\hat{u}_p^g+\delta_g-\bar{u}_p^g| \leq B\ (1-M_g)\\ \quad |\hat{u}_p^g+\delta_g-\underline{u}_p^g| \leq B\ (1-m_g)\\ \quad B\ (M_g-1)\leq \hat{u}_p^g + f_g\ \alpha - \bar{u}_p^g \leq B\ M_g\\ \quad B\ (m_g-1)\leq \underline{u}_p^g - \hat{u}_p^g - f_g\ \alpha \leq B\ m_g\\ (\lambda,u_c,e)\in\arg\min_{(\tilde{\lambda},\tilde{u}_c,\tilde{e})}\ \tilde{\lambda}\\ \textbf{s.t.}\begin{cases}-\tilde{\lambda}L_1 \leq M_\theta\ \theta + M_u\ \tilde{u}_c \leq \tilde{\lambda}L_1\\ u_c^{min}\ \tilde{e} \leq \tilde{u}_c \leq u_c^{max}\ \tilde{e}\\ \tilde{e}\leq R\ d \quad \tilde{e}\in\{0,1\}\end{cases}\end{cases}$$

where

- $\mathcal{G}$ denotes the set of generators devoted to the secondary load-frequency control
- λ is the minimum value which makes the lower-level problem feasible by multiplying componentwise the vector of limits L_1
- $\underline{u}_p$ and $\bar{u}_p$ are bounds on preventive actions
- s is the vector of uncertainty
- F denotes the active power flow balance in the post-contingency case
- B is a scalar positive big value
- t is a scalar discrete variable which establish an increase or a decrease of all the power output of generators
- f_g, δ_g denote respectively the maximum magnitude of the contribution of generator g to the secondary load-frequency control and its deviation from the active power setpoint $\hat{u}_p^g$
- M_g, m_g denote binary variables associated to the generator g. By fixing the variables M_g or m_g to one, the generator g can choose not to obey the secondary load-frequency control level α. When the variable M_g (respectively m_g) is equal to one, the power output $\hat{u}_p^g+\delta_g$ of generator g is fixed at $\bar{u}_p^g$ (respectively $\underline{u}_p^g$), in the remaining case the constraint $\delta_g = f_g\ \alpha$ is enforced
- L_1, L_2 are flow limits with $L_2 > L_1$
- R is a boolean matrix which links post-contingency violated constraints with specific corrective actions, associated to the binary variables e
- the matrices M_θ, M_u are introduced to express the calculated quantities corresponding to the security thresholds in the post-contingency state before corrective actions

The activation of corrective actions according to the system operator rules, managed by the binary decision variables d and p, operates as follows : as long as post-contingency flows $M_\theta\ \theta$ are below their limits L_1, from the last inequalities we have that $d=0$, which prevents any corrective action, as a consequence $u_c = 0$. Otherwise, if post-contingency flows $M_\theta\ \theta$ violate limits L_1 then $d=1$ and the corresponding corrective actions are activated (i.e. $u_c^{min} \leq u_c \leq u_c^{max}$) in order to bring flows below limits according to the inequalities $-L_1 \leq M_\theta\ \theta + M_u u_c \leq L_1$.

We compute the solution of the formulation (1) using an iterative algorithm for mixed-integer nonlinear bi-level programming proposed in Ref. [2], which is a generalization of Ref. [1] where x^u (respectively y^u) denotes a vector of continuous (respectively discrete) variables (same convention for x^l and y^l). In Ref. [4], we demonstrate through numerical simulations the feasibility of the method on very large systems and for a very large number of contingencies. However, the introduction of topological corrective actions (bus merging/splitting, line switching) at the lower level leads to adapt the configuration of the network to each uncertainty vector. Rather, we want to find a single topological action robust against all the uncertainties. Semi infinite programs (SIP) presented in the next section provide a relevant formulation to respect this requirement.

IV. Generalized semi-Infinite Programs including discrete variables at the upper and lower levels

Whenever an objective function value of (1) greater than one indicates the failure of PST actions to address the line overload problems, the resolution of an SIP program of the type :

$$\min_{x^u,y^u} f^U(x^u,y^u) \tag{2}$$

$$\textbf{s.t.}\quad g^U(x^u,y^u,x^l,y^l)\leq 0\ \ \forall (x^l,y^l): g^L(x^u,y^u,x^l,y^l)\leq 0$$

provides a topological corrective action (x^u, y^u) robust against all uncertainties (x^l, y^l) and the complexity of this action can be minimized through the definition of f^U in order to make it applicable. It is interesting to notice that the uncertainties of problem (1) become the lower variables of problem (2) and that the latter problem handles the corrective actions at the upper level. In Ref. [3], the authors explain how to deal with integer variables y^u and y^l. To evaluate the influence on flows of a topological action, the principle detailed in Ref. [5] is to calculate compensation injections y which reflect for a fictitious line either its disconnection or the fact that its reactance reaches zero (see the figure 3 below).

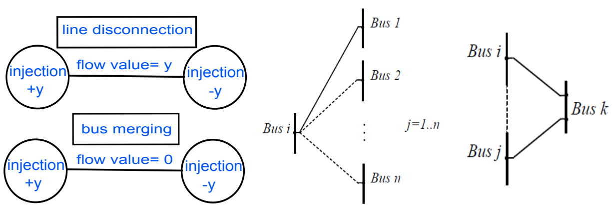

Fig. 3. Compensation injections and fictitious lines to simulate line disconnection/switching and bus merging/splitting

$$\min_{e_{ls}, u_c \in \{0,1\}, u_{ir}} \left\| u_c - u_c^{init} \right\|_1 + N_c \, \| e_{ir} \|_1 \qquad (3)$$

$$\textbf{s.t.} \begin{cases} u_{ir}^{min} \, e_{ir} \leq u_{ir} \leq u_{ir}^{max} \, e_{ir} \quad (injection\ removing) \\ \sum u_{ir} = 0 \\ |f_{init} + M_s s + M_y u_y + M_{ir} u_{ir}| \leq L_1 \ \forall (s, \delta, u_y, M, m, t) \\ \quad s.t. \\ \begin{cases} s \in S \ \ M, m, t \in \{0,1\} \\ \forall \ i \in \mathcal{I} \\ \quad |(f_{init} + M_s \, s + M_y \, u_y + M_{ir} \, u_{ir})_i| \leq B \, (1 - u_{ci}) \\ \quad |(f_{init} + M_s \, s + M_y \, u_y + M_{ir} \, u_{ir})_i - u_{y_i}| \leq B \, u_{ci} \\ \sum s = \sum_g \delta_g \quad (global\ balancing) \\ \underline{u}_p \leq \hat{u}_p + \delta \leq \bar{u}_p \\ \forall \ g \in \mathcal{G} \\ \quad B \, (t - 1) \leq \delta_g \leq B \, t \\ \quad |\delta_g - f_g \, \alpha| \leq B \, (M_g + m_g) \\ \quad |\hat{u}_p^g + \delta_g - \bar{u}_p^g| \leq B \, (1 - M_g) \\ \quad |\hat{u}_p^g + \delta_g - \underline{u}_p^g| \leq B \, (1 - m_g) \\ \quad B \, (M_g - 1) \leq \hat{u}_p^g + f_g \, \alpha - \bar{u}_p^g \leq B \, M_g \\ \quad B \, (m_g - 1) \leq \underline{u}_p^g - \hat{u}_p^g - f_g \, \alpha \leq B \, m_g \end{cases} \end{cases}$$

where

- $\mathcal{I}$ denotes the set of fictitious lines
- N_c is an integer value strictly greater than $\left\| u_c^{init} \right\|_1$
- the vector f_{init} denotes the initial flows determined from the initial injections p_{init} on the topology corresponding to all fictitious lines connected
- for each node k, $u_{ri}^{min}(k) \leq p_{init}(k) \leq u_{ri}^{max}(k)$. If $p_{init}(k) > 0$ then $u_{ri}^{min}(k) = p_{init}(k) = u_{ri}^{max}(k)$
- the matrices M_s, M_{ir} and M_y are introduced to express the dependence of active flows upon uncertainties, injection removing u_{ir} and dummy injections u_y related to bus merging/splitting
- x^u upper continuous variables = $\{u_{ir}\}$
- y^u upper discrete variables = $\{e_{ir}, u_c\}$
- x^l lower continuous variables = $\{\delta, s, u_y\}$
- y^l lower discrete variables = $\{M, m, t\}$

It should be noted that the introduction of injection removing in the formulation (3) provides a safeguard against a too conservative uncertainty set. Indeed, the problem (3) is always feasible, moreover the term weighted by N_c in the cost function ensures that no injection removing is used if a topological solution exists. To tackle semi-infinite problems of the form (2), Ref. [3] generates two finite sets of feasible points Y^{UBD} and Y^{LBD} in order to build convergent upper and lower bounding procedures. The essential steps algorithm are summarized on figure 4 below. To determine if an upper candidate $(\overline{x}^u, \overline{y}^u)$ satisfy all the constraints of the lower level associated to the set Y, we need to test the condition $g^{U,*}(\overline{x}^u, \overline{y}^u) \leq 0$ with the following definition :

$$g^{U,*}(\overline{x}^u, \overline{y}^u) = \max_{(x^l, y^l) \in Y} g^U(\overline{x}^u, \overline{y}^u, x^l, y^l)$$

$$\textbf{s.t.} \ \forall \ (x^l, y^l) \in Y : g^L(\overline{x}^u, \overline{y}^u, x^l, y^l) \leq 0$$

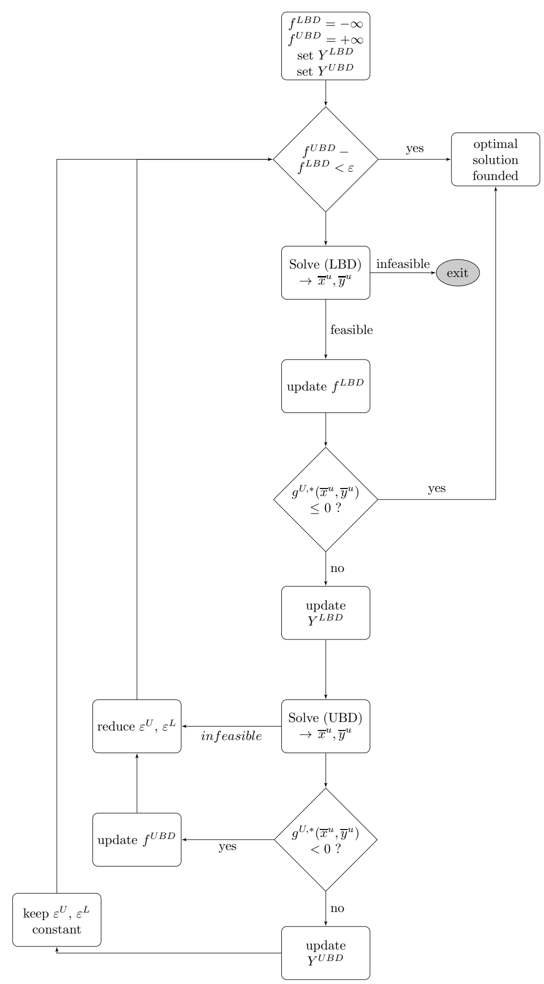

Fig. 4. Simplified flowchart of the algorithm proposed in [3].

where the concise notation

$$(x^u, y^u) \in Z(\varepsilon^U, \varepsilon^L, Y) \Leftrightarrow \boxed{\begin{array}{r} \varepsilon^U + g^U(x^u, y^u, x^l, y^l) \leq 0 \\ or \\ \varepsilon^L - g^L(x^u, y^u, x^l, y^l) \leq 0 \\ \forall\, (x^l, y^l) \in Y \end{array}}$$

enables to define the problems (LBD) and (UBD) :

- **(LBD)** $\min_{(x^u,y^u)\in Z(0,0,Y^{LBD})} f^U(x^u, y^u)$
- **(UBD)** $\min_{(x^u,y^u)\in Z(\varepsilon^U,\varepsilon^L,Y^{UBD})} f^U(x^u, y^u)$

This method was shown to be convergent under mild conditions without any convexity assumption.

V. Numerical simulations

The numerical simulations are based on three test cases :

- a real life 6726 bus system where the resolution of problem (1) aims to simulate on a large scale the effects of the outage of two generators under uncertainties including unexpected demand
- a modified IEEE 30 bus system with an initial topology leading to overloads, requiring corrective actions obtained from the resolution of problem (3)
- a small 12 bus system where the comprehensive list of the lines or transformers never affected by an overload is determined in the presence of large ranges of uncertainties which allow the removing or doubling of load.

A. 6726-bus system

A summary of the characteristics of this system according to the voltage level is given in Table I.

TABLE I
FRENCH EHV/HV NETWORK

voltage level (kV)	number of buses	number of lines
400	587	896
225	1405	1748
150	67	61
90	1149	1356
63	3174	3746
45	65	57
20	279	3

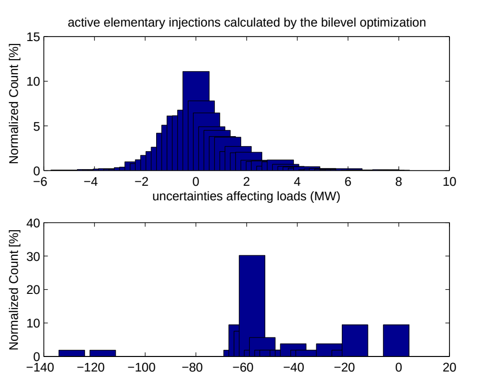

Fig. 5. Elementary injections after the tripping of two generator units

The problem to solve is made up of 29 PST and 145 generators (the specific treatment of uncertainties is detailed in section II). To emphasize the RSFP process, we chose to reduce to 53 the number of generators which obey to the secundary frequency control level. As a consequence, the limits of theses generators play a bigger part as can be seen in figure 5 above.

In the figure 6 below, the x-axis (respectively y-axis) represents the value of base case flows (respectively flows resulting from the resolution of problem (1)) expressed as a percent of the overload limits.

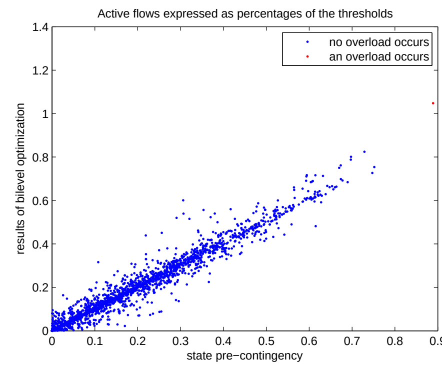

Fig. 6. Effect of uncertainties on flows post-contingency

The dispersion of the points along the first bisector indicates the combined effect of the tripping of two generator units which amounts to approximately 1646 MW and of the uncertainties given in figure 5. The points having their ordinate greater than one correspond to lines or transformers exceeding their transmission capacities. All generator units devoted to RSFP increase their productions but 48 of them contribute to the utmost of their capacity and do not comply with the positive variation of secondary load-frequency level as shown on the lower graph of figure 5. For this contingency, one should remark an increase of 3213 MW in the power generation provided by these 53 generator units. When the conventional analysis without uncertainties is carried out, this value falls to 1646 MW and the figure 7 below illustrates the absence of overloads.

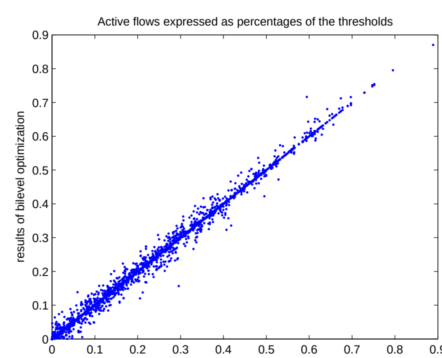

Fig. 7. Evolution of flows when no uncertainties are modeled

To sum up, the combination of two different factors, the occurrence of a contingency and the uncertainties surrounding the forecast elementary injections lead to the emergence of an significant overload (see the red point on the figure 6).

B. 30-bus system

We have modified the IEEE 30-bus system by adding three phase-shifting transformers (PSTs) as shown on the one-line diagram of Figure 8. Their location is inspired by Reference [8], which suggests that these are the optimal locations for placing a small number of PSTs in this system. The three (identical) PSTs have thus been installed in series with the lines originally defining the branches 15-18, 10-22 and 24-25. We supposed that their phase shift ranges for our illustration should be constrained to +/-20 degrees. Generator locations and limits were taken from references [6] and [7]. In our analysis, we suppose that the uncertainty is modeled by considering independent variations of loads in the range [-15%,+15%] at all the buses except for 10, 15, 19, 24. On the base case, these four buses have been splitted through the introduction of fictitious nodes 31, 32, 33, 34 and fictitious branches detailed in Table II below. When the uncertainty on the base case is combined to this initial topology, the resolution of problem (1) proves that the automatic action of PSTs is not sufficient to avoid line or transformer overloads : the flow in the line $16-17$ expressed in percentage terms of the threshold reaches 153%.

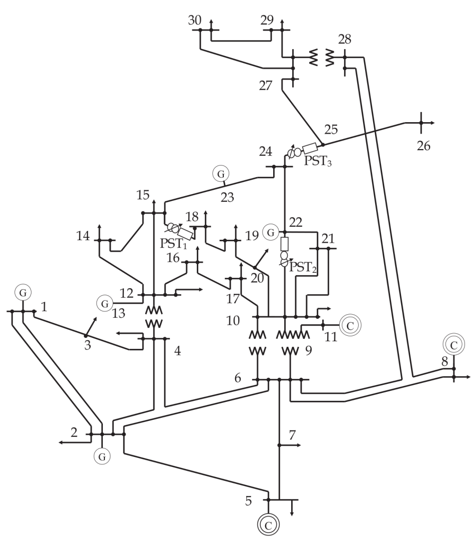

Fig. 8. Modified IEEE 30-bus system; adapted by adding three phase-shifting transformers

TABLE II
MODIFIED 30-BUS SYSTEM: TOPOLOGICAL ACTION FOUND

		modified branches	
solution $\mathbf{u_c}$	**fictitious branch**	**new definition**	**initial definition**
0	$15-32$	$32-23$	$15-23$
0	$19-33$	$33-20$	$19-20$
1	$24-34$	$34-25$	$24-25$
0	$10-31$	$31-20$	$10-20$
0	$10-31$	$31-17$	$10-17$
0	$10-31$	$31-21$	$10-21$
0	$10-31$	$31-22$	$10-22$

As shown in the above Table II, the topological solution of problem (3) consists in performing one merging at bus 24.

TABLE III
MODIFIED 30-BUS SYSTEM: GENERATORS

bus	$\underline{u}_p^g$	$\hat{u}_p^g$	$\bar{u}_p^g$	$\hat{u}_p^g+\delta_g$	$\hat{u}_p^g+f_g\ \alpha$
2	0	0.51655	0.8	0.678627	0.678627
23	0	0.267513	0.3	0.3	0.328292
27	0	0.336945	0.55	0.448373	0.448373

As indicated in table III, the total output of the three generator units subject to secondary power frequency control has increased by approximately 27 percent. This allows sharp and contrasting swings on the remaining nodal injections as can be seen from Table IV.

TABLE IV
MODIFIED 30-BUS SYSTEM: NODAL INJECTIONS AFFECTED BY UNCERTAINTIES

$variation-15\%$	:	$1, 13, 17, 18, 19, 20, 21, 22, 24, 26, 29, 30$
$variation+15\%$	:	$3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 16, 22$

C. 12-bus system

The small size of this network provides scope for study of very large uncertainty ranges. The key point is that these ranges may cover unintended topological changes as connection/disconnection, and by this way identify the weak points. To simplify, all the values of overload thresholds for lines and transformers are equal to 0.5 (p.u.), except for the lines between the generator units and the rest of the network where these are taken equal to 1.0 (p.u.). The one-line diagram of Figure 9 presents the results of problem (1) in terms on nodal uncertainties, which all have reached their limits except at node 11 due to the constraint of active power balance. The two generator units subject to secondary power frequency control are stretched to their maximum powers.

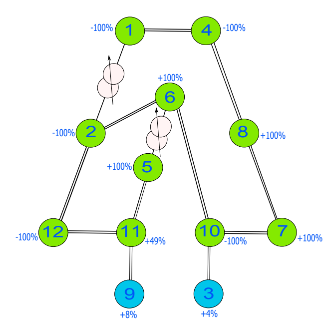

Fig. 9. 12-bus system with two generators (at nodes 9 and 3) and two PST

TABLE V
12-BUS SYSTEM: SUCCESSIVE ELIMINATION OF THE MORE STRINGENT OVERLOAD THRESHOLD

maximum ratio	**line or transformer**		**PST phase shifts (degrees)**	
λ	$name$	$limit$	PST_{6-5}	PST_{1-2}
1.49662	$10-6$	0.5	0.0	0.0
1.22979	$12-2$	0.5	-3.2	0.0
1.1702	$12-11$	0.5	-2.9	0.0

In the above Table V, the first line gives the most severe overload resulting from the uncertainties, one should remark that the conditions for activation of the two PSTs $6-5$ and $1-2$ are not fulfilled since their respective flows amount to 0.302044 and 0.375425, values that are below the limits. The next question is: what modification of thresholds is at least necessary to cope with such ranges of uncertainties? To determine these new thresholds, the successive elimination of the more stringent constraints in problem (1), one by one, is required. So at the total cost of three bilevel optimizations, we get the answer : multiply the thresholds of lines $10-6$, $12-2$ and $12-11$ by the coefficients 1.49662, 1.22979 and 1.1702. A last resolution of problem (1) is used to validate this proposition, the objective function becomes equal to one.

VI. Most appropriate algorithms

Problem (1) is a mixed-integer linear bilevel program. Early works on algorithms for this problem class include branch-and-bound algorithms Ref. [9]. More recently, cutting plane algorithms have been proposed Ref. [10], [11], [12]. Some of the published methods impose further restrictions such as only considering discrete variables. Furthermore, only slight changes in the model formulation may result in nonlinearities, making (1) a mixed-integer nonlinear bilevel program. For the nonlinear case, the authors in Ref. [13] propose a branch-and-bound algorithm that branches on both the upper and lower-level variables and respects the hierarchical nature of bilevel programs through specialized branching and bounding rules. In Ref. [2], a discretization algorithm for continuous nonlinear bilevel programs from Ref. [1], which in turn is based on Ref. [20], is extended to the mixed-integer case.

For problem (3), similarly to the solution of (1), we opt for a solution method that can in principle be applied to mixed-integer nonlinear generalized semi-infinite programs. In particular, we opt for methods that guarantee to generate feasible points in finite time. Based on Ref. [18], [19], the author of Ref. [14] proposes a method to solve generalized semi-infinite programs through interval extensions of the semi-infinite constraint on the lower-level variables. Based on Ref. [20], multiple methods have been proposed that consider a discretization of the lower level variables, which is populated successively by solutions of the lower-level program. In Ref. [16], a discretization of the lower-level variables combined with a so-called oracle problem is proposed to obtain a convergent bounding scheme. While convergence has been proven for the regular semi-infinite case Ref. [15], it has not for the generalized semi-infinite case. In Ref. [3], the authors propose the extension of the discretization algorithm from Ref. [17] from the semi-infinite case to the generalized semi-infinite case and prove convergence under comparatively weak assumptions. While the authors of Ref. [3] do not adress the presence of integer variables, Ref. [17] discusses the mixed-integer semi-infinite programs and the discussion is fully applicable to the generalized semi-infinite case.

VII. Conclusion

In this paper we have analyzed the task of dealing with uncertainties for security management of electric power systems assuming a pessimistic total power balance change. The problem faced by operation planning engineers is to examine critical situations which may include unexpected demand and the tripping of generators. The state-of-the-art of optimization methods enables to provide, through a 'max-min' resolution, a diagnosis of the day-ahead security, tractable for large scale power systems. When a potentially hazardous contingency is detected, the next challenge is how to provide topological actions robust against all uncertainties to eliminate overloads. To achieve this objective, we have used a semi-infinite programming formulation including an exact penalization of injection removing which makes the problem always feasible. This approach is promising on academic test cases but it remains to be proven on a real life network that a small number of proven effective elementary topological actions can be combined in an automatic way. A great value for future improvements is that robust optimization has the ability to give useful results by considering only restricted parts of a network, reflecting uncertainty with boundary through injection ranges at the corresponding nodes.

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