Affine Arithmetic Formulation of the Unit Commitment Problem Under Uncertainty

David Romero-Quete
Department of Electrical and Electronics Engineering
Universidad Nacional de Colombia, Bogota, Colombia
Email: dfromeroq@unal.edu.co

Claudio Canizares
Department of Electrical and Computer Engineering
University of Waterloo, Ontario, Canada, N2L3G1
Email: ccanizar@uwaterloo.ca

***Abstract*—This paper proposes a method based on affine arithmetic (AA) to solve the unit commitment (UC) problem, considering load and renewable energy (RE) uncertainties. The main idea is to formulate an AA-based constrained multi-objective problem that not only provides a robust commitment solution, but also provides confidence intervals for active powers and operating costs, as well as dispatch solutions for all possible load and RE generation realizations for the predetermined uncertainty bounds. Moreover, the AA-based UC (AAUC) approach allows to explore solutions where the impact of the re-dispatch cost in the total operation cost can be reduced by adjusting the objective function weight values. The proposed approach can be used to better estimate day-ahead energy prices and explore more cost-efficient solutions under uncertainty, as well as for real-time dispatching. The AAUC approach is tested and compared against stochastic optimization UC (SOUC), interval optimization UC (IOUC), and Monte Carlo simulation (MCS) approaches, on a six-bus system and a modified IEEE 118-bus system. The simulations results show that the proposed approach provides more accurate confidence intervals for active power and operating costs than IOUC, using MCS results as the benchmark, and has a better computational performance than SOUC.**

***Index Terms*—Affine arithmetic, interval optimization, multi-objective optimization, stochastic optimization, uncertainty, unit commitment.**

I. Introduction

uncertainty in power systems has increased in recent years due to the further integration of Renewable Energy (RE) sources, demand response programs, and electric-vehicle charging stations in the power grid. In order to reduce the impacts of these uncertainties on system operation, power system operators can make use of tools such as Unit Commitment (UC) to determine the most cost-efficient schedule of generating units, which satisfies the system constraints (e.g. power balance constraints, reserve constraints, transmission capacity limits) and unit constraints (e.g. maximum/minimum capacities, ramping up/down rate limits, minimum on/off time requirements) [1]–[4], as well as yield the adequate dispatchable generation to best hedge the system against these uncertainties, as discussed in detail in this paper.

In Deterministic UC (DUC), load and renewable generation are estimated by forecasting, while uncertainty is addressed through reserve constraints which ensure enough generation to achieve a reliable operation. However, since uncertainties are not directly taken into account, this method can produces too conservative or infeasible UC-dispatch solutions, due to an overestimation or an underestimation of the uncertainty in the forecast. In this context, methods such as scenario based stochastic optimization, robust optimisation, and interval optimisation have been proposed in the literature for UC under uncertainty.

In the stochastic optimization UC (SOUC) approach [5]–[8], the objective is to minimize the expected operating cost with uncertainty being represented by a set of scenarios, which are generated based on probability distribution functions (pdfs) of the random variables (e.g. load, RE generation). The main drawbacks of this approach are the need to accurately identify pdfs, and the fact that feasibility is not guaranteed for all the uncertainties, especially when scenario reduction techniques are employed to decrease the computational burden [9].

The robust optimization UC (ROUC) approach [10]–[13] aims to find the optimal UC which minimize the worst-case cost to safeguard the system against all realization of uncertainty in a chosen range. However, this technique yields conservative solutions for the chosen uncertainty interval, as it optimizes for the worst scenario, which is a shortcoming of this method [14]. A non-conservative robust UC formulation is proposed in [15], where the base-case scenario is optimized instead of the worst-case scenario, while ensuring feasibility for all possible realization within the chosen uncertainty range.

In the interval optimization UC (IOUC) formulation [16], [17], uncertainty is represented by confidence interval numbers without considering pdfs. The objective in this approach is to minimize the operation cost of the base case, or central forecast, while ramping-constraints between the central forecast and the lower and upper bounds of two adjacent operating periods are imposed, in order to ensure that the UC is feasible for any scenario within the uncertainty confidence interval. However, as demonstrated in this paper, the confidence intervals obtained for the active power dispatched by each generator, based on optimistic and pessimistic scenarios, tend to be too conservative compared to the actual realization.

Other studies reported in the literature have proposed the combination of the aforementioned methods in order to overcome some of their limitations. For instance in [18], the stochastic and robust approaches are combined using a weighted objective function which considers the costs of both approaches; this technique provides less conservative solutions than ROUC and more robust solutions than SOUC.

In order to achieve a balance between operating cost and robustness, in [19], SOUC is implemented for the first few hours of the optimization horizon, while IOUC is used for the remaining hours. In [20], a hybrid Markovian and interval optimization approach to solve the transmission-constrained UC is proposed; in this case, uncertainty is modeled by states instead of scenarios, reducing the complexity of the problem when compared to scenario-based formulations.

Recently, Affine Arithmetic (AA), a self-validated computing technique, has been proposed for addressing uncertainty in power systems [21]–[24]. The techniques proposed in these papers are based on the property that AA allows to keep track of the accuracy of the computed quantities without requiring information about the type of uncertainty in the parameters [25]; thus, pdfs are not required to obtained accurate and robust solutions. Therefore, in this paper, a transmission-constrained UC formulation is proposed based on the AA-based computing framework proposed in [24], which allows the formulation of a generic mathematical programing problem under uncertainty, based on a set of AA-based minimization, equality, an inequality operators. The main idea is to simultaneously minimize the base-case scenario cost and the corrective dispatch cost associated with the uncertainties, by solving a multi-objective optimization problem with continuous variables and constraints modeled in an AA domain.

The main objectives and contributions of the current paper can be summarized as follows.

- A novel UC solution approach based on AA is proposed, which not only provides a robust UC solution, but also provides robust ED solutions for all possible load and RE generation realizations for predetermined uncertainty bounds.
- The proposed approach is compared to IOUC and SOUC techniques, demonstrating that the AA-based UC (AAUC) approach provides more accurate confidence intervals for active power and operating costs than IOUC, using Monte Carlo Simulation (MCS) as the benchmark, as well as more robust and computationally more efficient solutions than SOUC.

The rest of this paper is organized as follows: Section II briefly reviews the main AA definitions and operators of the AA-based computing framework, as well as introducing the DUC, SOUC and IOUC formulations. Section III discusses the proposed AAUC model. In Section IV, a 6-bus system and a modified IEEE 118-bus benchmark system are used as case studies to asses and compare the performance of the proposed method. Finally, Section V summarizes the main conclusions and contributions of this paper.

II. Background

A. Elements of AA

AA is a range analysis technique introduced in [26], which handles both external (e.g. imprecise or missing input data, uncertainty in the mathematical modeling) and internal (e.g. round off and truncation errors) uncertainty sources. AA is similar to standard Interval Mathematics (IM) [27], but this paradigm provides narrower bounds in the computing process by keeping track of correlations between the input and the computed quantities [28]. In AA, each uncertain variable χ has an affine representation $\hat{\chi}$, as follows:

$$\hat{\chi} = \chi_0 + \chi_1\varepsilon_1 + \chi_2\varepsilon_2 + ... + \chi_p\varepsilon_p \tag{1}$$

where χ_0 is the central value of $\hat{\chi}$; ε_h, known as noise symbols, are p symbolic real variables assumed to be unknown but bounded in the interval [-1,1], which represents an independent component of the total uncertainty of the variable χ; and χ_h are the coefficients defining the magnitude of the corresponding uncertainty components. One noise symbol can contribute to the uncertainty of multiple quantities (e.g. inputs, outputs) resulting from the evaluation of an expression; this sharing of noise symbols results in some partial dependency between two affine forms $\hat{\chi}$ and $\hat{\psi}$, determined by the coefficients χ_h and ψ_h [28].

In order to perform AA computations, it is necessary to replace the elementary real-number operators by equivalent mappings between affine forms. For linear functions, the corresponding affine extension is obtained by expanding and rearranging only the noise symbols characterising the affine forms $\hat{\chi}$ and $\hat{\psi}$. However, if the mapping is non-linear, the corresponding affine extension cannot be described by an affine combination of the "primitive" noise symbols ε_h. Therefore, in this case, it is necessary to identify an affine function, which approximates the function reasonably well over its domain. The reader is referred to [28] and [29] for further information of the definition of affine and non-affine operations.

B. AA-based Constrained Optimization Problems

A new theoretical framework to solve uncertain optimization problems based on AA is introduced in [24]. More specifically, this framework aims to solve the following non-linear constrained optimization problem under data uncertainties represented as affine forms:

$$\begin{array}{llll} \min\limits_{\hat{z}} & \hat{f}(\hat{z}) & & \\ \text{s.t.} & \hat{g}_j(\hat{z}) & = & 0 \quad \forall j \in \mathcal{J} \\ & \hat{h}_k(\hat{z}) & < & 0 \quad \forall k \in \mathcal{K} \end{array} \tag{2}$$

where $\hat{z} = (\hat{z}_1, ..., \hat{z}_{N_z})$ represents the unknown affine form of the state variables, which include both dependent and control variables; $\hat{f}$ is the affine, continuous and differentiable function describing the problem objectives; and $\hat{g}_j$ and $\hat{h}_k$ are continuous and differentiable affine functions representing j^{th} equality and k^{th} inequality constraints, respectively, in the respective sets $\mathcal{J}$ and $\mathcal{K}$. To solve this optimization problem, an extension into the affine domain of the minimization operator and the main comparison operators $<, >, \leq, \geq$, and $=$ is proposed in [24]. The definition of these operators is presented next, since they are important for the purpose of the current paper. Further information about the derivation of these operators can be found in [30].

Definition 1 (Similarity operator for affine forms $\overset{A}{\approx}$): Two affine forms $\hat{\chi} = \chi_0 + \sum_{h=1}^{p+p_{na}} \chi_h \varepsilon_h$ and $\hat{\psi} = \psi_0 + \sum_{h=1}^{p+p_{na}} \psi_h \varepsilon_h$ are similar with an approximation degree $Ł_{\chi,\psi}$, i.e. $\hat{\chi} \overset{A}{\approx} \hat{\psi}$, if and only if:

$$\{\chi_h = \psi_h \quad \forall h \in (0,...,p)\} \wedge \left\{ Ł_{\chi,\psi} = \sum_{h=p+1}^{p+p_{na}} |\chi_h| + |\psi_h| \right\} \tag{3}$$

where $\varepsilon_{p+1},...,\varepsilon_{p+p_{na}}$ are the noise symbols describing the endogenous uncertainties, which are generated by approximations of p_{na} non-affine functions (e.g. multiplications and trigonometric functions); $\chi_{p+1},...,\chi_{p+p_{na}}$ are the partial deviations representing the upper-bound of the corresponding approximation errors, which can be computed by using the expressions derived from the Chebyshev Theorem [28].

Definition 2 (Inequality operator for affine forms $\overset{A}{<}$): Given two affine forms $\hat{\chi} = \chi_0 + \sum_{h=1}^{p_\chi} \chi_h \varepsilon_h$ and $\hat{\psi} = \psi_0 + \sum_{h=1}^{p_\psi} \psi_h \varepsilon_h$, then $\hat{\chi} \overset{A}{<} \hat{\psi}$, if and only if:

$$\chi_0 + \sum_{h=1}^{p_\chi} |\chi_h| < \psi_0 - \sum_{h=1}^{p_\psi} |\psi_h| \tag{4}$$

Definition 3 (Operator $\underset{A}{\min}$): Given a differentiable, non-linear function $f : \Re \rightarrow \Re$ and the affine form $\hat{\chi} = \chi_0 + \sum_{h=1}^{p} \chi_h \varepsilon_h$, then the following AA-based minimization problem:

$$\underset{\underset{\hat{\chi}}{A}}{\min} \quad \hat{f}(\hat{\chi}) = f_0(\hat{\chi}) + \sum_{h=1}^{p} f_h(\hat{\chi})\varepsilon_h + \sum_{h=p+1}^{p+p_{na}} f_h(\hat{\chi})\varepsilon_h \tag{5}$$

is equivalent to the following deterministic multi-objective programming problem:

$$\min_{(\chi_0,\chi_1,...,\chi_p)} \left\{ f_0(\chi_0,\chi_1,...,\chi_p), \sum_{h=1}^{p+p_{na}} |f_h(\chi_0,\chi_1,...,\chi_p)| \right\} \tag{6}$$

Based on (3), (4) and (6), the optimization problem (2) can be solved by solving the following deterministic multi-objective constrained problem:

$$\begin{aligned} \min_{\hat{z}} \quad & \left\{ f_0(\hat{z})), \sum_{h=1}^{p+p_{na}} |f_h(\hat{z})| \right\} \\ \text{s.t.} \quad & \hat{g}_j(\hat{z}) \overset{A}{\approx} 0 && \forall j \in \mathcal{J} \\ & \hat{h}_k(\hat{z}) \overset{A}{<} 0 && \forall k \in \mathcal{K} \end{aligned} \tag{7}$$

C. Deterministic Transmission-Constrained UC Formulation [31]

The deterministic transmission-constrained UC is a mixed-integer linear programming (MILP) problem, whose objective function is given as:

$$\min_{P_{i,t},x_{i,t},SU_{i,t},SD_{i,t}} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} \Big[F_i(P_{i,t})x_{i,t} + C_i^{sdn} SD_{i,t} + C_i^{sup} SU_{i,t} \Big] \tag{8}$$

where $P_{i,t}$ is the output power of unit i in the set of dispatchable generators $\mathcal{I}$, at time t in the time set $\mathcal{T}$; $x_{i,t}$ is the scheduled state ON/OFF of unit i at time t; $SD_{i,t}$ and $SU_{i,t}$ are binary variables indicating the shut-down and start-up decision, respectively; and C_i^{sdn} and C_i^{sup} are the shut-down and start-up cost of unit i, respectively. The function $F_i(.)$ accounts for the cost of each thermal unit using a piecewise linear upper approximation of the convex cost curve.

The transmission-constrained UC is subject to the following constraints :

$$\begin{aligned} &\sum_{i\in\mathcal{I}_b} P_{i,t} + W_{b,t} - \sum_{\{b,m\}\in\mathcal{L}|m>b} B_{bm}(\delta_{b,t} - \delta_{m,t}) \\ &+ \sum_{\{b,m\}\in\mathcal{L}|m<b} B_{mb}(\delta_{b,t} - \delta_{m,t}) = D_{b,t} \quad \forall t \in \mathcal{T}, b \in \mathcal{B} \end{aligned} \tag{9}$$

$$P_i^{min} x_{i,t} \le P_{i,t} \le P_i^{max} x_{i,t} \quad \forall t \in \mathcal{T}, i \in \mathcal{I} \tag{10}$$

$$P_{i,t} - P_{i,t-1} \le R_i^{up} \quad \forall t \in \mathcal{T}, i \in \mathcal{I} \tag{11}$$

$$P_{i,t-1} - P_{i,t} \le R_i^{dn} \quad \forall t \in \mathcal{T}, i \in \mathcal{I} \tag{12}$$

$$-P_{l,bm}^{max} \le B_{bm}(\delta_{b,t} - \delta_{t,m}) \le P_{l,bm}^{max} \quad \forall t \in \mathcal{T}, \{b,m\} \in \mathcal{L} \tag{13}$$

$$\delta^{min} \le \delta_{b,t} \le \delta^{max} \quad \forall t \in \mathcal{T}, b \in \mathcal{B} \tag{14}$$

$$SU_{i,t} - SD_{i,t} = x_{i,t} - x_{i,t-1} \quad \forall t \in \mathcal{T}, i \in \mathcal{I} \tag{15}$$

$$SU_{i,t} + SD_{i,t} \le 1 \quad \forall t \in \mathcal{T}, i \in \mathcal{I} \tag{16}$$

$$x_{i,t} = g_i^{on/off} \quad \forall t \le (L_i^{up,min} + L_i^{dn,min}), i \in \mathcal{I} \tag{17}$$

$$\sum_{tt=t-g_i^{up}+1} SU_{i,tt} \le x_{i,t} \quad \forall t \ge L_i^{up,min} \tag{18}$$

$$\sum_{tt=t-g_i^{dn}+1} SD_{i,tt} \le 1 - x_{i,t} \quad \forall t \ge L_i^{dn,min} \tag{19}$$

where constraints (9) are the power balance equations for each node b in the set of buses $\mathcal{B}$; $W_{b,t}$ is the RE power available at bus b at time t; B_{bm} is the admittance of the line connecting buses b and m in the set of lines $\mathcal{L}$; $\delta_{b,t}$ is the voltage angle at bus b at time t; $D_{b,t}$ is the load at bus b at time t; and $\mathcal{I}_b$ is the subset of dispatchable generators connected to bus b. RE curtailment is not included in (9), since there is not incentive considered here for RE power spillage, which is the case in most markets (e.g. Ontario, Canada). Constraints (10) corresponds to minimum P_i^{min} and maximum P_i^{max} generation capacity limits of controllable units. Constraints (11)-(12) impose the ramp-up R_i^{up} and ramp-down R_i^{dn} rates limits of the dispatchable generators. Constraints (13) and (14) are the constraints of the linear dc power flow, which guarantee that the maximum line flow limits $P_{l,bm}^{max}$ and angles limits δ^{min} and δ^{max} are respected, respectively. Constraints (15) and (16) associate the unit commitment decisions, $SD_{i,t}$ and $SU_{i,t}$, with the status variable $x_{i,t}$, as well as ensure that each unit is not turned-on and -off simultaneously. Finally, constraints (17)-(19) enforce the minimum up-time g_i^{up} and the minimum down-time g_i^{dn}, where $L_i^{up,min}$ and $L_i^{dn,min}$ are the number of periods that unit i is required to stay ON or OFF at the beginning of the optimization horizon, and $g_i^{on/off}$ is the status of unit i at $t=0$.

D. Stochastic Optimization [17]

In SOUC, a set of scenarios $\mathcal{S}$ is used to model the uncertainty of the random variables (e.g. RE generation, load). In this case, the following objective function is used to minimize the base-case cost plus the expected corrective dispatch cost of scenarios, while respecting constraints (9)-(19) for each scenario:

$$\min_{P_{i,t},x_{i,t},SU_{i,t},SD_{i,t}} \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\left[C_i^{sdn}SD_{i,t}+C_i^{sup}SU_{i,t}\right] + \sum_{s\in\mathcal{S}}\pi_s\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}F_i(P_{i,t,s})x_{i,t} \quad (20)$$

where π_s is the probability of a scenario s in the set of scenarios $\mathcal{S}$ for a given pdf. Typically, decompositions techniques such as Bender's decomposition [17] and Lagrangian relaxation [32] are used to solve this problem when it is applied to large-scale power systems.

E. Interval optimization [16]

IOUC minimizes the operating base-case cost (8) while respecting constraint (9)-(19) for three scenarios: central forecast (cf), lower bound (lb) and upper bound (ub). Additionally, the following ramping-constraints are imposed in order to ensure that the UC is feasible for any scenario within the predetermined uncertainty confidence interval:

$$P_{i,t,ub}-P_{i,t-1,cf}\le R_i^{up} \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \quad (21)$$

$$P_{i,t-1,cf}-P_{i,t,lb}\le R_i^{dn} \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \quad (22)$$

$$P_{i,t,ub}-P_{i,t-1,lb}\le R_i^{up} \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \quad (23)$$

$$P_{i,t-1,ub}-P_{i,t,lb}\le R_i^{dn} \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \quad (24)$$

III. AA-based Unit Commitment Formulation

A. Affine Forms

Load and RE generation uncertainties can be represented by affine forms as follows:

$$\hat{D}_{b,t}=D_{0,b,t}+\sum_{n=1}^{p_d}D_{n,b,t}\varepsilon_n \quad \forall t\in\mathcal{T}, b\in\mathcal{B} \quad (25)$$

$$\hat{W}_{b,t}=W_{0,b,t}+\sum_{r=1}^{p_w}W_{r,b,t}\varepsilon_r \quad \forall t\in\mathcal{T}, b\in\mathcal{B} \quad (26)$$

where $D_{0,b,t}$ and $W_{0,b,t}$ are the forecasted values of load and RE at node b at time t; $D_{n,b,t}$ and ε_n are the partial deviations and noise symbols representing the load forecasting errors, respectively; $W_{r,b,t}$ and ε_r are the partial deviations and noise symbols representing the RE forecasting errors, respectively; and p_d and p_w are the number of noise symbols used to describe load and RE uncertainties, respectively. Note that the affine forms $\hat{D}_{b,t}$ and $\hat{W}_{b,t}$ do not share any noise symbols, since the uncertainty sources for the load and the RE generation are assumed to be independent. The number of noise symbols and the values of the partial deviations of the affine forms in (25) and (26) can be obtained by a characterization of the statistical properties of the random variables [24], considering aspects such as temporal and spacial correlation among uncertainties.

Based on (25) and (26), the continuous variables of the transmission-constrained UC, i.e. generator active power, voltage angles, and line power flows, can be represented by the following affine forms:

$$\hat{P}_{i,t}=P_{0,i,t}+\sum_{n=1}^{p_d}P_{n,i,t}\varepsilon_n+\sum_{r=1}^{p_w}P_{r,i,t}\varepsilon_r \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \quad (27)$$

$$\hat{\delta}_{b,t}=\delta_{0,b,t}+\sum_{n=1}^{p_d}\delta_{n,b,t}\varepsilon_n+\sum_{r=1}^{p_w}\delta_{r,b,t}\varepsilon_r \quad \forall t\in\mathcal{T}, b\in\mathcal{B} \quad (28)$$

$$\hat{P}_{l,t}=P_{0,l,t}+\sum_{n=1}^{p_d}P_{n,l,t}\varepsilon_n+\sum_{r=1}^{p_w}Pl_{r,l,t}\varepsilon_r \quad \forall t\in\mathcal{T}, l\in\mathcal{L} \quad (29)$$

where the first term of each affine form is the central value of the variable, the second term is the deviation of the variable because of the load forecasting errors, and the third term is the deviation of the variable because of the RE generation forecasting errors.

B. Problem Statement

The main idea of the AAUC approach is to find a UC schedule and the parameters of the affine forms (27)-(29), which simultaneously minimize the base-case scenario cost and the corrective dispatch cost, while satisfying all the units and system constraints. In this context, based on *Definition 3* and using the weighted-sum multi-objective optimization method, it is proposed here that the objective function of the AAUC problem be formulated as follows:

$$\min_{\hat{P}_{i,t},x_{i,t},SU_{i,t},SD_{i,t}} \sum_{t}\sum_{i}\Bigg[w[F_{0,i}(\hat{P}_{i,t})x_{i,t} + C_i^{sdn}SD_{i,t} + C_i^{sup}SU_{i,t}] + (1-w)\sum_{h=1}^{p+p_{na}}|F_{h,i}(\hat{P}_{i,t})|x_{i,t}\Bigg] \quad (30)$$

where $p = p_d + p_w$, and $w \in [0,1]$ represents the weight related to the objective of minimizing the operation cost of the base case or affine central value of the function cost, which is the cost of the dispatch for the central forecast $F_{0,i}$ plus the commitment cost; hence, $1-w$ is the weight related to the objective of minimizing the re-dispatch cost $F_{h,i}$ or affine radius of the function cost. The value of w can be chosen according to the decision maker's degree of conservativeness with respect to the uncertainties; thus, a value of w close to 1 leads to cost-effective solutions that are sensitive to the uncertainties, while a value of w close to 0 leads to solutions that are more expensive but less sensitive to the uncertainties.

Constraints (9)-(14) can be formulated in affine form based on (3) and (4). If RE curtailment is included in (9), a new affine form representing this variable must be included in the formulation. Constraints (15)-(19) remain the same, since they are only related to the binary variables, which are "crisp" variables, without an affine form representation. Note that the resulting mathematical model has absolute values in the objective function and constraints, which can be easily linearized with additional variables and linear constraints [33], so that the optimization problem becomes an MILP problem that can be readily solved by using commercial solvers such as CPLEX.

C. Model Outputs

Except for the commitment decisions, all the outputs of the model (e.g. operating cost, dispatched active power, line power flows) are represented by affine forms. These affine forms can be transformed either into interval representation or into deterministic solutions for particular realizations of the random variables. Thus, an affine form $\hat{\chi}$ can be converted into interval representation by using the following operators:

$$\overline{\Delta}(\hat{\chi}) := \chi_0 + \sum_{h=1}^{p}|\chi_k| \quad (31)$$

$$\underline{\Delta}(\hat{\chi}) := \chi_0 - \sum_{h=1}^{p}|\chi_k| \quad (32)$$

where $\overline{\Delta}(\hat{\chi})$ and $\underline{\Delta}(\hat{\chi})$ are the upper bound and lower bound of the affine form, respectively, forming the interval $[\underline{\Delta}(\hat{\chi}), \overline{\Delta}(\hat{\chi})]$. The intervals for the active power dispatched by each generator and for the operating price can be useful for applications such as energy pricing, reserve pricing, and day-ahead market clearing.

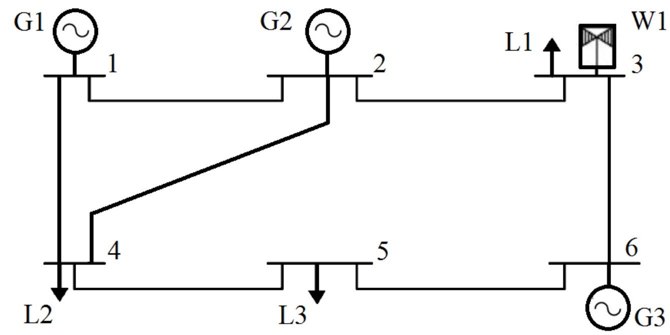

Fig. 1. Comparison of the UC-dispatch solutions for different values of the objective function weight w for the 6-bus system.

A specific value in the interval [-1,1] can be assigned to the noise symbols of the output affine forms, if there is a certain degree of certainty in the RE and load forecast (e.g. forecasts with 5-min time resolution). This results in a feasible economic dispatch solution for the robust commitment solution, which can be used for real-time economic dispatch.

IV. Numerical Results

To test and validate the proposed AAUC approach, a six-bus system [6] and a modified IEEE 118-bus system are used. The GAMS 23.3.3 environment [34], and the CPLEX 12.1.0 optimization engine [35] were used to implement all UC formulations, with a minimum relative MIP gap of 10^{-4}. Simulation were performed on an Intel® Xeon® CPU L7555 1.87GHz 4-processor server.

The 6-bus system is used to: (i) illustrate the effect of the value of the objective weight w on the UC and dispatch solutions; (ii) compare the AAUC approach against the DUC, SOUC, and IOUC approaches; (iii) compare the dispatched active power bounds for the AAUC and IOUC approaches with those obtained with MCS, which are used as the benchmark; and (iv) study the impact of uncertainties levels in the AAUC solution. The results obtained for the modified IEEE 118 bus benchmark system are used to evaluate the computational performance of the proposed UC method.

A. 6-bus System

The 6-bus system shown in Fig.1 has load and wind power generation profiles defined in [17]. The total load is assumed to be distributed as 20%, 40%, and 40% for Buses 3, 4, and 5, respectively. Tolerance bounds of the load and wind power forecasting errors are assumed as ±10% and ±30%, respectively.

(i) Objective Function Weight Analysis: Figure 2 shows the UC-dispatch solutions obtained for $w = 0.9$ and $w = 0.1$; only the affine central values are plotted to facilitate visualization. When $w = 0.9$, the most important objective is to minimize the affine central value of the operating cost, in which case most of

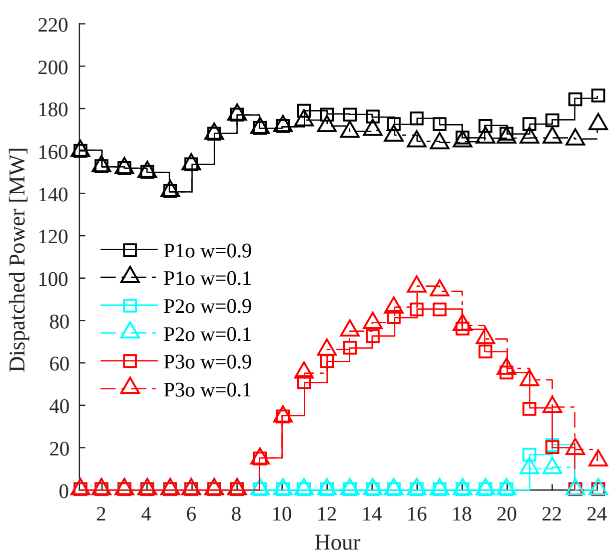

Fig. 2. Comparison of the UC-dispatch solutions for different values of the objective function weight w for the 6-bus system.

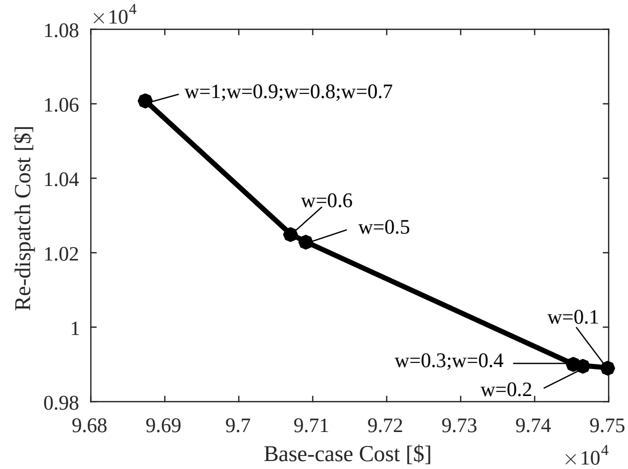

Fig. 3. Pareto front from the AAUC formulation for the 6-bus system.

the demand is supplied by the cheapest generator, Unit 1 (black plots), in order to reduce the base-case operating cost. On the other hand, when $w = 0.1$, the main objective is to reduce the variation of the operation cost due to the forecasting errors of load and RE generation; consequently, the power dispatched for Unit 3 is bigger, in order to allow Unit 1 to supply the power variations due to forecasting errors at a lower cost, while respecting its ramp constraints, thus resulting in a bigger but less variable operating cost.

Figure 3 shows the Pareto front obtained for different values of w. Distinct values of w may give the same UC-dispatch solutions because of the discrete variables in the problem. System operators can chose a solution belonging to the Pareto front based on their degree of conservativeness. The solution for $w = 0$ is not included in Fig. 3, since it leads to an expensive dispatch that does not make sense for real applications.

(ii) Comparison of Various Approaches: Table I shows the day-ahead operating cost for the DUC, the SOUC, the IOUC, the AAUC with $w = 0.9$, and the MCS, based on 1000 scenarios of load and wind for a uniform pdf, which represents the most extreme case of uncertainty, and as per the convergence of the MCS expected value. Observe that the DUC provides the least expensive but least robust solution for the base-case or central forecast, since it does not take into account possible deviations in the forecasted values. The SOUC provides a less expensive solution for the base-case than the IOUC and the AAUC formulations, but it only guarantees the feasibility for the set of scenarios considered during the optimization process, while the IOUC and the AAUC provide robust UC solutions immunized against all the possible scenarios in the uncertainty tolerance bounds considered on the problem formulation. The AAUC yields the biggest base-case operating cost, providing more accurate intervals than the IOUC, compared to the MCS solution which is taken as the benchmark, since the proposed AAUC approach simultaneously minimize the affine central value and the affine radius of the operating cost, thus providing tighter operating cost bounds than the IOUC.

Figure 4 shows the UC-dispatch solutions obtained from the SOUC, IOUC, and AAUC formulations. The generator commitment solution is the same for all the formulations, but the power dispatch solution differs from one method to another. The main difference, as mentioned before, is that the AAUC formulation provides a robust UC and minimizes the effect of the random variables on the operating cost, which results in a reduction of the power supplied by the cheapest unit (Unit 1), in order to allow corrective dispatch actions at a lowest price, while respecting the unit constraints.

Based on (27)-(29), note that the AAUC also provides dispatch solutions for all possible load and RE generation realizations belonging to the predetermined uncertainty bounds. For example, Table II shows a dispatch corresponding to a particular load and wind power realization; this dispatch was computed by adjusting the values of the load ($\varepsilon_1, \varepsilon_2, \varepsilon_3$) and wind power ($\varepsilon_4$) noise symbols of the affine forms:

$$\hat{P}_{1,12} = 177,48 + 3,85\varepsilon_1 + 7,7\varepsilon_2 + 7,7\varepsilon_3 - 0,08\varepsilon_4 \quad (33)$$

$$\hat{P}_{3,12} = 60,68 + 0,97\varepsilon_1 + 1,94\varepsilon_2 + 1,94\varepsilon_3 - \varepsilon_4 \quad (34)$$

which are the affine forms obtained with the AAUC for the power dispatch at hour 12 for Unit 1 and Unit 3, respectively; thus computing a feasible dispatch solution without the need for solving the optimization problem associated with the particular realization.

(iii) Power Bounds Comparison: Figure 5 shows the dispatched active power confidence intervals for Unit 1 and Unit 3 obtained by MCS, IOUC, and the AAUC with $w = 0.9$. Notice that for the first eight periods where only Unit 1 is committed, the different approaches provide similar power intervals, with a slight difference due to the approximations in the optimization

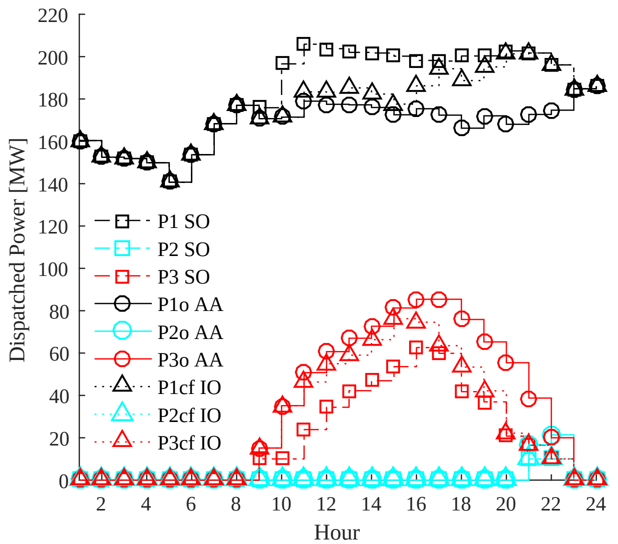

Fig. 4. UC-dispatch solutions for SOUC, IOUC, and AAUC formulations for the 6-bus system.

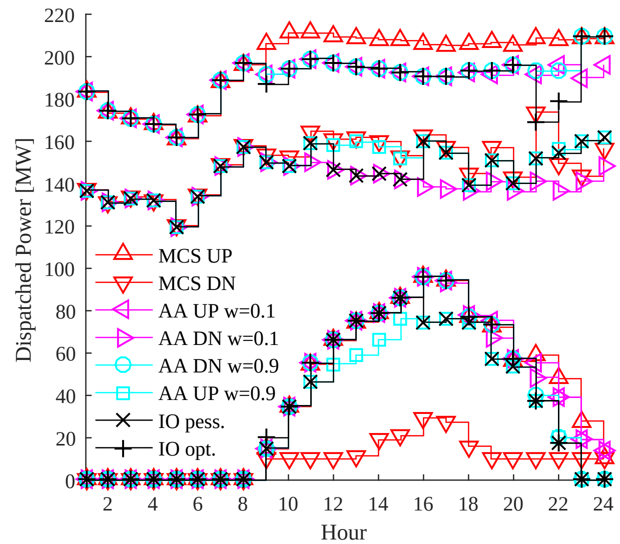

Fig. 5. Comparison of AAUC, IOUC and MCS intervals

TABLE I
OPERATING COST FOR DIFFERENT UC FORMULATIONS FOR THE 6-BUS SYSTEM.

	Base-case Cost [$]	Lower Cost [$]	Upper Cost [$]
DUC	93,205	-	-
SOUC	95,123	-	-
IOUC	95,463	86,723	108,695
AAUC	96,872	86,263	107,482
MCS	93,943	89,068	98,374

TABLE II
DISPATCH FOR A PARTICULAR LOAD AND WIND POWER REALIZATION

Realization					Dispatch	
Random Variable	Central Forecast [MW]	Actual Realiza. [MW]	Error [%]	Noise Symbol Value	Unit	Power [MW]
$D_{4,12}$	48.23	50.64	5	0.5	$P_{1,12}$	180.12
$D_{5,12}$	96.46	104.18	8	0.8	$P_{2,12}$	0
$D_{6,12}$	96.46	89.71	7	-0.7	$P_{3,12}$	60.7
$W_{3,12}$	3	3.6	20	0.67		

TABLE III
OPERATING COST FOR DIFFERENT TOLERANCE BOUNDS OF LOAD FORECASTING ERRORS

Tolerance Bound	Base-case Cost [$]	Lower Cost [$]	Upper Cost [$]
5%	94,487	88,571	100,403
10%	96,872	86,263	107,482
15%	105,309	88,327	122,291
20%	Inf.	Inf.	Inf.

solution procedures. Observe that for the other hours, the proposed AAUC approach for $w = 0.9$ tends to be closer to the MCS results than IOUC, especially for hours 12 to 15, and 21 to 22; however, there is an important difference between the intervals provides by MCS and those from both the AAUC and the IOUC, especially for Unit 3, since both IOUC and AAUC methods enforces the feasibility of the ramp constraints between different scenarios. Finally, note that for the last two hours, MCS shows that for some realizations of the random variables, Unit 3 should be committed to obtain cheaper solutions, which is similar to the solution obtained with the AAUC approach for $w = 0.1$, where Unit 3 is committed, unlike the solutions obtained with IOUC, SOUC or AAUC for $w = 0.9$, thus better capturing these less likely events.

(iii) Sensitivity Analysis: Figure 6 shows the UC-dispatch solutions obtained for different tolerance bounds of wind power forecasting errors; in this case, w and the tolerance bound of load forecasting errors are set as 0.9 and 10%, respectively. Notice that the presumed wind uncertainty interval can affect both commitment and dispatch solutions, providing different degrees of conservativeness depending on the width of the presumed tolerance bounds of the forecasting errors. Table III shows the day-ahead operating cost for different presumed load uncertainty intervals; in this case w and the tolerance bound of wind power forecasting errors are set as 0.9 and 30%, respectively. When the presumed load uncertainty interval is set as 20%, the problem is infeasible since the systems cannot deal with these possible load deviations. For a load forecasting error tolerance bound of 15%, note that the lower bound of the operating cost is bigger than when the tolerance bound is equal to 10%; this is because the commitment solution of the first case is more conservative than the second case, resulting in more expensive solutions, even for scenarios with low demand.

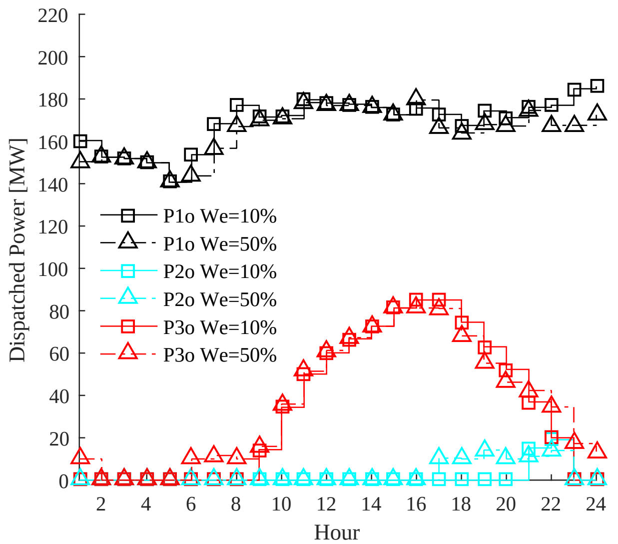

Fig. 6. UC-dispatch solutions for different tolerance bounds of wind power forecasting errors

B. 118 bus system

In this section, results are presented of simulations for a modified IEEE 118-bus benchmark system [17], which has 54 thermal units, three wind farms, and 186 branches, whose detailed data and load profiles can be found at [36]. Wind power generation profiles of the 3 wind farms are the same as in [37]. Tolerance bounds of the load and wind power forecasting errors are both set to 10%, for all the loads and wind farms. Different tolerance bounds for each load and wind farm of the system may be chosen, if available, without significantly affecting the computational performance.

Table IV shows the results obtained for different values of the objective weight w. For this study case, the minimum cost for the worst-case is obtained when $w = 0.5$, since $1,762,018 + 184,471 = 1,946,489$. As expected, the cheapest base-case solution is obtained for $w = 0.9$; however, this solution leads to the biggest worst-case cost $(1,746,362 + 217,938 = 1,964,300)$. Moreover, it can be seen that the value of w not only affects the dispatch decisions but also the commitment decisions, as shown in the third column of the table, where the total commitment hours of all generators, as a measure of reserve, are listed; for small values of w, the thermal units are committed for more hours, since these solutions are more conservative. These results show that the AAUC approach gives sufficient decision elements to the system operator to choose a UC-dispatch solution to operate the system, according to the system characteristics and the desired risk level. Finally, in the last column of the Table IV, it can be seen that the value of w also affects the solution computing time. For some values of w the computing time is small enough for real-time applications, as for $w = 0.2$ or $w = 0.7$; however, there are some values, such as $w = 0.1$ or $w = 0.9$, for which the computing time could be too large for real-time applications, since these take more than 1h. Nevertheless, more efficient implementations and optimization solvers should reduce these times.

TABLE IV
RESULTS FOR DIFFERENT VALUES OF w FOR THE 118-BUS SYSTEM.

w	Central Value of the Operating Cost [$]	Radius of the Operating Cost [$]	Total UC [h]	Comp. time [s]
0.9	1,746,160	217,938	705	17,055
0.8	1,747,353	212,557	702	1,300
0.7	1,748,648	208,530	707	182
0.6	1,760,492	186,257	715	1,574
0.5	1,762,018	184,471	714	1,850
0.4	1,781,766	170,429	730	3,829
0.3	1,798,108	161,086	733	1,138
0.2	1,802,157	159,589	732	98
0.1	1,802,799	159,536	737	4,958

TABLE V
RESULTS FOR DIFFERENT UC FORMULATIONS FOR THE 118-BUS SYSTEM.

	Base Case Cost [$]	Upper Cost [$]	Total UC [h]	Comp. time [s]
DUC	1,739,680	-	663	17
SOUC (10 scen.)	1,741,086	-	663	214
SOUC (50 scen.)	1,740,470	-	663	9,450
SOUC (100 scen.)	1,740,214	-	663	58,163
IOUC	1,743,708	2,014,882	697	160
AAUC (w=0.9)	1,746,160	1,966,710	705	17,055
AAUC (w=0.2)	1,802,157	1,961,745	732	98

Table V shows the results obtained for the DUC, the SOUC, the IOUC, and the AAUC approaches. Simulations were carried out with 10, 50, and 100 scenarios for the SOUC, to show the computing time explosion of the SOUC. In general, IOUC has a better computational performance than SOUC and AAUC, since it just considers 3 scenarios; however, for $w = 0.2$, the AAUC shows better performance than IOUC. For $w = 0.9$, the AAUC presents its worst computational performance (4h:44min:15s); however, this time is still smaller than the time for the basic SOUC formulation with 100 scenarios.

V. CONCLUSIONS

This paper has proposed a novel UC formulation based on an AA-based computing framework. The approach was shown to simultaneously minimize the base-case scenario costs and the corrective dispatch costs. Compared with existing approaches which only provide a finite set of power dispatch solutions, the proposed approach provided dispatch solutions for all possible load and RE generation realizations belonging to the predetermined uncertainty bounds.

The results show that this new approach provides more accurate operating cost and power dispatch intervals than the IOUC approach, since it includes the re-dispatch cost in the objective function. Also, it is shown that the solution conservativeness depended on the presumed tolerance bounds of the uncertainties. Finally, the AAUC approach is shown to have better computational performance than the classical SOUC approach.

REFERENCES

[1] J. Shaw, “A direct method for security-constrained unit commitment,” *IEEE Trans. Power Syst.*, vol. 10, no. 3, pp. 1329–1342, 1995.

[2] M. Shahidehpour, H. Yamin, and Z. Li, *Security-constrained unit commitment*. Wiley-IEEE Press, 2002, pp. 275–310.

[3] N. Padhy, “Unit commitment - a bibliographical survey,” *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 1196–1205, 2004.

[4] X. Guan, P. Luh, H. Yan, and J. Amalfi, “An optimization-based method for unit commitment,” *Int. J. Electr. Power Energy Syst.*, vol. 14, no. 1, pp. 9–17, feb 1992.

[5] Jianhui Wang, M. Shahidehpour, and Zuyi Li, “Security-constrained unit commitment with volatile wind power generation,” *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1319–1327, aug 2008.

[6] L. Wu, M. Shahidehpour, and T. Li, “Stochastic security-constrained unit commitment,” *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 800–811, 2007.

[7] P. a. Ruiz, C. R. Philbrick, E. Zak, K. W. Cheung, and P. W. Sauer, “Uncertainty management in the unit commitment problem,” *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 642–651, may 2009.

[8] P. Carpentier, G. Gohen, J.-C. Culioli, and A. Renaud, “Stochastic optimization of unit commitment: a new decomposition framework,” *IEEE Trans. Power Syst.*, vol. 11, no. 2, pp. 1067–1073, may 1996.

[9] Y. Dvorkin, Y. Wang, H. Pandzic, and D. Kirschen, “Comparison of scenario reduction techniques for the stochastic unit commitment,” in *Proc. IEEE PES General Meeting*, July 2014, pp. 1–5.

[10] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, “Adaptive robust optimization for the security constrained unit commitment problem,” *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52–63, feb 2013.

[11] C. Zhao, J. Wang, J.-P. Watson, and Y. Guan, “Multi-stage robust unit commitment considering wind and demand response uncertainties,” *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2708–2717, aug 2013.

[12] R. Jiang, J. Wang, and Y. Guan, “Robust unit commitment with wind power and pumped storage hydro,” *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 800–810, may 2012.

[13] R. Jiang, M. Zhang, G. Li, and Y. Guan, “Two-stage network constrained robust unit commitment problem,” *Eur. J. Oper. Res.*, vol. 234, no. 3, pp. 751–762, 2014.

[14] H. Ye, Y. Ge, M. Shahidehpour, and Z. Li, “Uncertainty marginal price, transmission reserve, and day-ahead market clearing with robust unit commitment,” *IEEE Trans. Power Syst.*, vol. 8950, no. c, pp. 1–1, 2016.

[15] H. Ye and Z. Li, “Robust security-constrained unit commitment and dispatch with recourse cost requirement,” *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 3527–3536, sep 2016.

[16] Y. Wang, Q. Xia, and C. Kang, “Unit commitment with volatile node injections by using interval optimization,” *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1705–1713, aug 2011.

[17] L. Wu, M. Shahidehpour, and Z. Li, “Comparison of scenario-based and interval optimization approaches to stochastic SCUC,” *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 913–921, may 2012.

[18] C. Zhao and Y. Guan, “Unified stochastic and robust unit commitment,” *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3353–3361, aug 2013.

[19] Y. Dvorkin, S. Member, H. Pandžić, M. A. Ortega-vazquez, and D. S. Kirschen, “A hybrid stochastic / interval approach to transmission-constrained unit commitment,” *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 621–631, 2015.

[20] Y. Yu, P. B. Luh, E. Litvinov, T. Zheng, J. Zhao, and F. Zhao, “Grid integration of distributed wind generation: Hybrid markovian and interval unit commitment,” *IEEE Trans. Smart Grid*, vol. 6, no. 6, pp. 3061–3072, nov 2015.

[21] A. Vaccaro, C. A. Canizares, and D. Villacci, “An affine arithmetic-based methodology for reliable power flow analysis in the presence of data uncertainty,” *IEEE Trans. Power Syst.*, no. 2, pp. 624–632, may 2010.

[22] J. Munoz, C. A. Canizares, K. Bhattacharya, and A. Vaccaro, “An affine arithmetic-based method for voltage stability assessment of power systems with intermittent generation sources,” *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4475–4487, nov 2013.

[23] A. Vaccaro, C. A. Canizares, and K. Bhattacharya, “A range arithmetic-based optimization model for power flow analysis under interval uncertainty,” *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1179–1186, may 2013.

[24] A. Vaccaro and C. Canizares, “An affine arithmetic-based framework for uncertain power flow and optimal power flow studies,” *IEEE Trans. Power Syst.*, vol. 8950, no. c, pp. 1–15, 2016.

[25] J. Stolfi and L. H. de Figueiredo, “An introduction to affine arithmetic,” *Tendências em Matemática Apl. e Comput.*, vol. 4, no. 3, pp. 297–312, 2003.

[26] J. Stolfi and L. H. D. Figueiredo, “Self-validated numerical methods and applications,” in *Proc. of the Monograph for 21st Brazilian Mathematics Colloquium, Citeseer*, 1997.

[27] R. E. Moore, *Methods and Applications of Interval Analysis*, ser. Studies in Applied and Numerical Mathematics. Society for Industrial and Applied Mathematics, 1979.

[28] L. H. de Figueiredo and J. Stolfi, “Affine arithmetic: Concepts and applications,” *Numer. Algorithms*, vol. 37, no. 1-4, pp. 147–158, Dec 2004.

[29] A. Vaccaro, C. Pisani, and A. Zobaa, “Affine arithmetic-based methodology for energy hub operation-scheduling in the presence of data uncertainty,” *IET Gener. Transm. Distrib.*, vol. 9, no. 13, pp. 1544–1552, 2015.

[30] A. Vaccaro, “Affine arithmetic for power and optimal power flow analyses in the presence of uncertainties,” Ph.D. dissertation, University of Waterloo, 2015.

[31] Rajan, D. and Takriti, S., “Minimum up/down polytopes of the unit commitment problem with start-up costs,” IBM Research Rep., 2005.

[32] A. Papavasiliou and S. S. Oren, “Multiarea stochastic unit commitment for high wind penetration in a transmission constrained network,” *Oper. Res.*, vol. 61, no. 3, pp. 578–592, 2013.

[33] “Absolute Values”. [Online]. Available: http://lpsolve.sourceforge.net/5.1/absolute.htm

[34] R. E. Rosenthal, *GAMS - A user's guide*, 2016. [Online]. Available: http://www.gams.com/help/topic/gams.doc/userguides/GAMSUsersGuide.pdf

[35] *GAMS - The solver manuals*, 2016. [Online]. Available: http://www.gams.com/help/topic/gams.doc/solvers/allsolvers.pdf

[36] Illinois Institute of Technology. ROSCUC_118. [Online]. Available: http://motor.ece.iit.edu/Data/ROSCUC_118.xls

[37] B. Hu and L. Wu, “Robust SCUC with load and wind uncertain intervals,” in *Proc. IEEE PES Gen. Meet.*, July 2014, pp. 1–5.