On Statistical Size and Placement of Generation and Load For Synthetic Grid Modeling*

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***Abstract*- This paper investigates the problem of generation and load size and placement when developing a synthetic power grid model. Both electrical parameters and topology measures are considered. Previous studies have indicated that the relative location of generation and load buses in a realistic grid are not random but correlated and an entropy based optimization approach was developed to determine a set of correlated siting parameters for generation and load buses. Using the exponential distribution of individual generation capacity and load settings, and the non-trivial correlation between the generation capacity or load setting together with the nodal degree of a generation or load bus we develop an approach to generate a statistically correct random set of generation capacities and load settings. We then assign them to generation and load buses in a synthetic grid.**

***Index terms*- Synthetic Power Grid Modeling, Generation Setting, Load Setting, and Statistical Analysis.**

I. Introduction

Synthetic power grid models, which are entirely fictitious but are required to have the same topology and electrical statistics of a realistic power grid, will help address the urgent need of grid data faced by many power system researchers and scientists, i.e., to provide sufficient case studies without disclosing the sensitive information associated with a real power grid.

To accomplish the goal of developing synthetic networks, extensive research has been conducted in order to expose salient grid related properties and to collect the statistics of grid topologies and electrical parameters needed to develop some useful models [1-18]. Reference [14] provides a comprehensive study on geographical approaches for creating synthetic power grid models. It describes several structural statistics and uses them to present a methodology for generating synthetic line topologies with realistic parameters.

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In [18] the authors propose a systematic methodology to augment the synthetic network base case for energy economic studies. Here the cost model for generators is determined based on the fuel type and generation capacity. This model can be used for electricity market and power system operation analysis.

As mentioned in [15], a valid synthetic grid model needs to include at least the following critical components: (a) the electrical grid topology which is fully defined by a grid admittance matrix; (b) the generation and load settings which indicate their correlated placement and sizing; (c) the transmission constraints which include, among other things, the capacity limits of both transmission lines and transformers.

This paper presents our recent study results on the statistics of generation capacities and placement in a synthetic grid modeling. A set of approaches has been developed to generate a statistically correct random set of generation capacities and assign them to the generation buses in a synthetic grid according to the approximate scaling function of total generation capacity versus network size, the estimated exponential distribution of individual generation capacities, the non-trivial correlation between the generation capacity and the nodal degree of a generation bus. The proposed approaches may be readily applied to determining the load settings in a synthetic grid model and to studying the statistics of the flow distribution and to estimating the transmission constraint settings.

The main contributions of this paper are summarized as follows:(1) a set of statistical analysis on the generation capacities and its correlation with topology metrics is presented; (2) a statistical approach is proposed to determine the generation capacity at any generation bus in a synthetic grid modeling which takes into account both topology and electric measures; (3) a statistical-based approach is presented to determine the load at any load bus considering both topology and electric measures.

The rest of the paper is organized as follows: Section II presents an approach to system modeling of a power grid; Section III provides a statistical analysis of generation capacities and their correlation with nodal degrees. This section also presents an algorithm for creating and assigning generation capacities to generation buses; Section IV describes a statistically-based algorithm to determine load size at any load bus. Finally, Section V concludes the paper and discusses future work.

II. *System Modeling*

The electrical topology of a power grid, with N buses and M branches which represent transmission lines and transformers in a high-voltage transmission network, is fully described by an admittance matrix $Y_{N\times N}$, which is defined as

$$Y = A^T\Lambda^{-1}(z_l)A, \quad (1)$$

where A is the branch-node incidence matrix of size ($M \times N$), defined as $A_{li} = 1$; $A_{lj} = -1$; if the l^{th} branch is from node i to node j, $A_{lk} = 0$, for $k \neq i, j$; with $l = 1, \ldots, M$. $\Lambda^{-1}(.)$ denotes the diagonal inverse matrix with a specific vector and z_l the vector of branch impedances. Therefore, the so-called DC power flow distribution in a grid follows its network constraints as:

$$P(t) = B'(t)\theta(t) \quad (2)$$

$$F(t) = \Lambda(y_l)A\theta(t) \quad (3)$$

where $P(t) = [P_g(t), -P_L(t), P_C]^T$ represents the vector of injected real power from generation, load, and connection buses. Obviously, the power injection from connection buses equals zero, i.e., $P_C = 0$. $y_l = 1/x_l$ represents the vector of the branch admittances ignoring the resistive component. $\theta(t)$ is the vector of phase angles, and $F(t)$ the vector of real-power delivered along the transmission branches. Besides the network constraints, grid operation also needs to account for the constraints of generation capacity, load settings, and transmission capacity such as

$$P_g^{Min} \le P_g \le P_g^{Max} \quad (4)$$

$$P_L^{Min} \le P_L \le P_L^{Max} \quad (5)$$

$$F^{min} \le F \le F^{Max} \quad (6)$$

From what is presented above it is clear that the dynamics of a power grid not only depend on the "electrical" topology but also the generation and load settings including their locations and sizing. The placement of generation and load is equivalent to bus type assignments which are addressed in [15] - [16]. In this paper we expand our previous work to determine size and placement of installed generators and loads.

III. *The Statistics of Generation Size*

Generation siting means to determine both the placement and electrical size of each generation unit. Our previous study [11] indicated that the relative location of generation and load buses in realistic grids are not random but are correlated. And an entropy based optimization approach was proposed as a procedure to determine a set of correlated sitings for generation and load. In this paper we focus on the problem of determining the electrical size of generation units, i.e., the maximum capacity of each generator.

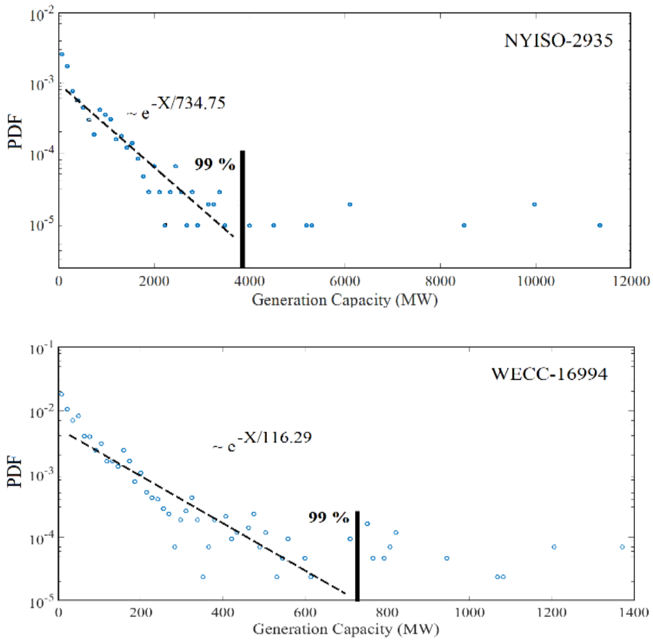


Fig. 1: Empirical PDF of generator capacities in NYISO-2935 and WECC-16994 bus system

In this section we first examine the statistical features of generation capacities in realistic power grids in terms of aggregate generation capacity, distribution of individual capacities, and their non-trivial correlation with nodal degrees.

In [17], we derived a scaling function for aggregate generation capacity in a grid versus its network size:

$$\log P_g^{tot}(N) = -0.21(\log N)^2 + 2.06(\log N) + 0.66 \quad (7)$$

where N is the network size, $P^{tot} = \sum_{n=1}^{N_g} P_{g_n}^{Max}$ denotes the total generation capacity, N_g is the total number of generation buses, and the logarithm is with base 10. Equation (7) implies that the total generation capacity in a grid tends to grow as a power function when the network size is small. However, as the network size becomes larger, the scaling curve begins to bend down and grow slower.

We observe that the statistical distribution of generation capacity and demand within a power grid based on realistic grid data such as the NYISO-2935, and the WECC-16994 systems, shows that more than 99% of the generation units (and the loads as well) follow an exponential distribution with about 1% having extremely large capacities (or demand) falling outside of the normal range defined by the expected exponential distribution indicated by the empirical probability density function (PDF) of generation capacities shown in Fig. 1. A possible cause of these distribution exceptions may come from an inherent heavy tailed distribution or result from boundary equalization due to network reduction.

After studying the scaling property and distribution of generation capacities, we examine the correlation between the generation capacities and other topology metrics. For the purpose of statistical analysis and algorithm development for generation capacity generation and assignment, we define the two normalized variables

$$\overline{P_{g_n}^{Max}} = P_{g_n}^{Max} / \max_i P_{g_i}^{Max}, \tag{8}$$

$$\overline{k_n} = k_n / \max_i k_i. \tag{9}$$

Both variables assume values in the interval [0, 1]. The statistics derived from the data of a number of realistic grids indicate that there exists a significant correlation between the nodal degree of a generation bus and its capacity as evidenced by a Pearson coefficient of $\rho\left(\overline{P_{g_n}^{Max}}, \overline{k_n}\right) \in [0.25, 0.5]$. Fig. 2 shows scatter plots of normalized generation capacity versus the normalized nodal degree for the NYISO-2935 and WECC-16994 systems. Note that they exhibit similar distribution patterns. That is, most data points are densely located within the region of $\overline{P_{g_n}^{Max}} \in [0, 0.2]$ and $\overline{k_n} \in [0, 0.5]$, while very few are located in the region of $\overline{P_{g_n}^{Max}} \geq 0.6$.

When two variable, say $\overline{P_{g_n}^{Max}}$ and $\overline{k_n}$, are considered, we may put them together to define a point $(\overline{P_{g_n}^{Max}}, \overline{k_n})$ in two-dimensional space. The density function $f(\overline{P_{g_n}^{Max}}, \overline{k_n})$, when integrated on a set A gives the probability of the event that the value of $(\overline{P_{g_n}^{Max}}, \overline{k_n})$ is in the set A. That is,

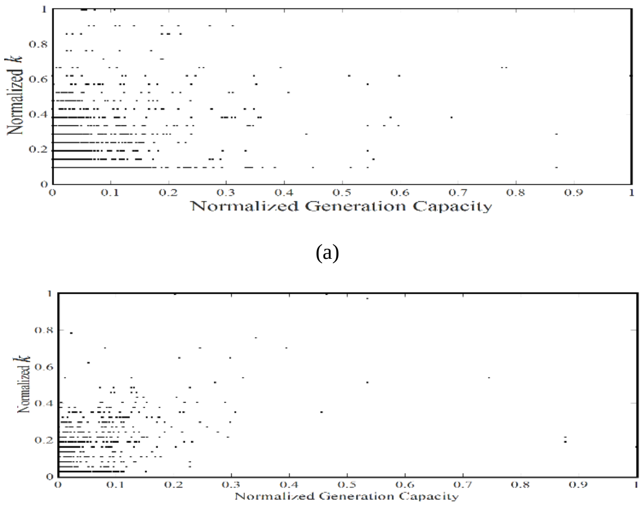
$$\Pr(A) = \Pr\left(\left(\overline{P_{g_n}^{Max}}, \overline{k_n}\right) \in \mathrm{A}\right) \tag{10}$$

Fig. 3 illustrates the 2-D empirical probability density function (PDF) of normalized node degree versus normalized generation capacity for the WECC-16994 bus system. Based on the this empirical PDF, a two- dimensional probability distribution table such as that shown in Table I for the WECC-16994 bus system can be formulated. The data in the table is useful for algorithm development for assigning the generation capacity values to each generation bus discussed in the next section.

III.A Assigning generation capacities to generation buses.

In this section we introduce an approach for generating a statistically correct random set of generation capacities and assigning them to generation buses according to the approximate scaling function of total generation capacity versus network size, the estimated exponential distribution of individual generation capacities, and the correlation between generation capacity and the nodal degree of a generation bus.

Given a synthetic grid topology with N buses among which N_g buses have generation units, we may determine the



(a)

(b)

Fig. 2. Scatter plots of normalized node degree versus normalized generation capacities (a) WECC-16994 (b) NYISO-2935 bus system

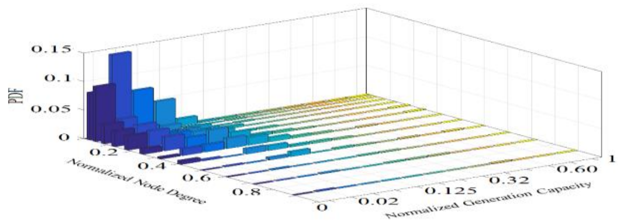


Fig. 3. 2-D empirical PDF of normalized node degree versus normalized generation capacity in WECC-16994 bus system.

aggregate generation capacity $P_g^{tot}(N)$ using equation (7) and generate a statistically correct random set of N_g generation capacities which follows an exponential distribution of generation capacities with 1% of generated capacities switched to super large values. Some scaling adjustment may be necessary in order to preserve the same aggregate generation capacity given by $P_g^{tot}(N)$. Next, the following algorithm is used to assign the generation capacities to each generation bus with respect to the statistical pattern represented by the data presented in Table I.

Step 1: Estimate the total generation capacity P_g^{tot} using (7).

Step 2: Generate a statistically correct random set of generation capacities $[P_{g_n}^{Max}]_{1\times N_g}$. It should be noted that 99 % of generated capacities follow the exponential distribution and remaining one percent is guaranteed to take supper large values (2~3 times greater than all generation capacities which follow the exponential distribution).

Step 3: Scale the generated capacities if $\sum_{n=1}^{N_g} P_{g_n}^{Max} > 1.05P^{tot}$ to ensure the aggregate generation capacity remains in the range specified by $P_g^{tot}(N)$. The scaling function is given as:

Table I. Probability analysis of normalized node degree and normalized generation capacity in WECC-16994 bus system

	$\overline{k_n}$ →	0.00–0.01	0.01–0.03	0.03–0.06	0.06–0.1	0.1–0.15	0.15–0.21	0.21–0.28	0.28–0.36	0.36–0.45	0.45–0.55	0.55–0.66	0.66–0.78	0.78–1.00	Marginal Prob
$\overline{P_{g_n}^{Max}}$	1.00–0.78	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.002
	0.78–0.66	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.004
	0.66–0.55	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.003
	0.55–0.45	0.000	0.001	0.000	0.004	0.008	0.000	0.002	0.001	0.000	0.001	0.000	0.001	0.000	0.018
	0.45–0.36	0.006	0.002	0.000	0.009	0.008	0.003	0.003	0.002	0.000	0.000	0.000	0.000	0.000	0.034
	0.36–0.28	0.003	0.011	0.012	0.017	0.013	0.007	0.003	0.002	0.000	0.001	0.000	0.000	0.000	0.072
	0.28–0.21	0.009	0.024	0.016	0.024	0.013	0.004	0.003	0.001	0.000	0.000	0.000	0.000	0.001	0.097
	0.21–0.15	0.025	0.027	0.016	0.013	0.009	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.097
	0.15–0.1	0.027	0.031	0.010	0.010	0.005	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.088
	0.1–0.06	0.033	0.017	0.003	0.003	0.005	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.063
	0.06–0.03	0.090	0.030	0.01	0.008	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.151
	0.03–0.01	0.082	0.140	0.070	0.04	0.010	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.360
	0.01–0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Marginal Prob		0.283	0.291	0.147	0.141	0.077	0.022	0.017	0.008	0.001	0.003	0.000	0.001	0.002	1.000

$$[P_{g_n}^{Max}]'_{1\times N_g} = [P_{g_n}^{Max}]_{1\times N_g} \times \frac{P_g^{tot}}{\sum_{n=1}^{N_g} P_{g_n}^{Max}} \quad (11)$$

where $[P_{g_n}^{Max}]'_{1\times N_g}$ is the updated generation capacities.

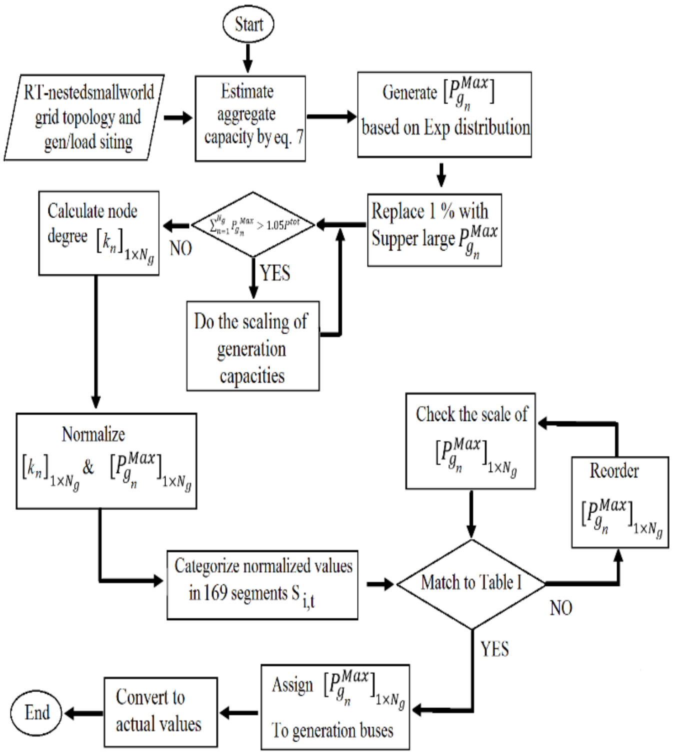


Fig. 4. Algorithm flowchart to assign random generation capacities to generation buses

Step 4: Calculate the node degree for all generation buses 1 to N_g based on the topology information of given synthetic power grid.

Step 5: Normalize both generation capacities and node degrees and categorize them evenly into 169 square regions with specific range of $\overline{P_{g_n}^{Max}}$ and $\overline{k_n}$.

Step 6: Check the similarity with Table I and reorder the mismatched segments.

Step 7: Assign the generated capacities to nominated generation buses with respect to their node degrees.

Step 8: Convert the normalized values to actual values.

The flowchart of the proposed algorithm is depicted in Fig. 4.

IV. *The Statistics of Load setting*

This section introduces a statistically–based approach to generating a set of static loads and assign them to the load buses. In this section we first investigate the statistical features of static loads in realistic power grids in terms of total demand, distribution of individual loads, and their correlation with nodal degrees.

In [17] we reported the results of our analysis on the scaling function of total demand in a grid versus its network size. The scaling of aggregate demand can be represented as a function of network size as

$$\log P_L^{tot}(N) = -0.20(logN)^2 + 1.98(\log N) + 0.58 \quad (12)$$

where $P_L^{tot}(N) = \sum_{n=1}^{N_L} P_{L_n}$ denotes the total generation capacity and N_L is the total number of load buses. The results showed that in realistic power networks the total demand tends to grow as a power function. It is important to point out that in a given grid topology, although the total demand can be fully determined by the presented scaling function, failure to maintain a balance between total load and resources causes frequency to vary from its target value. Thus, it is crucial to consider both scaling function and aggregate sources to achieve a reasonable value for total demand.

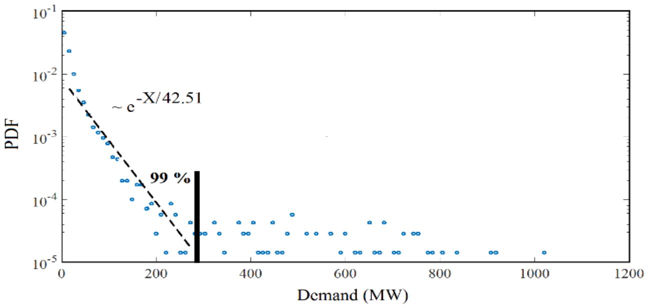


Fig. 5. Empirical PDF of loads in WECC-16994 bus system

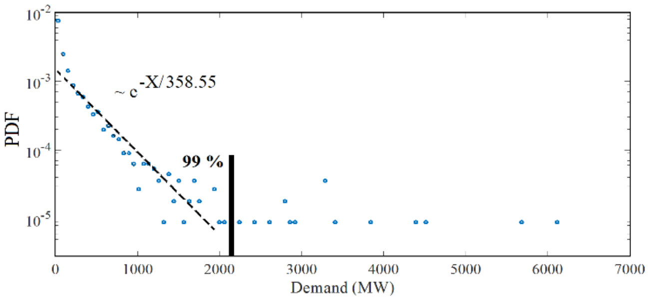


Fig. 6. Empirical PDF of loads in NYISO-2935 bus system

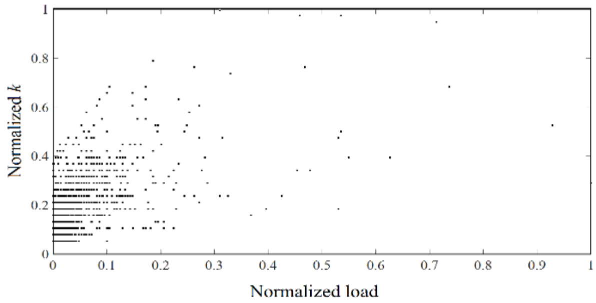


Fig. 7. Scatter plot of normalized node degree versus normalized load for the WECC-16994 bus system

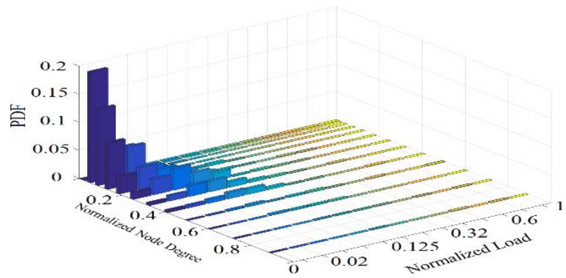


Fig. 8. The 2-D empirical PDF of normalized loads versus normalized node degrees in WECC-16994 bus system.

Table II. Probability Distribution of normalized node degree and normalized load in WECC-16994 bus system

		$\overline{k_n}$													Marginal Prob
		0.00 / 0.01	0.01 / 0.03	0.03 / 0.06	0.06 / 0.1	0.1 / 0.15	0.15 / 0.21	0.21 / 0.28	0.28 / 0.36	0.36 / 0.45	0.45 / 0.55	0.55 / 0.66	0.66 / 0.78	0.78 / 1.00	
$\overline{P_L}$	1.00 / 0.78	0	0	0	0	0	0.001	0	0	0	0.001	0	0.001	0	0.002
	0.78 / 0.66	0	0	0	0	0.001	0.001	0.001	0.001	0	0.001	0	0.001	0	0.003
	0.66 / 0.55	0	0	0	0.002	0.002	0.002	0.002	0	0	0	0	0	0	0.008
	0.55 / 0.45	0	0.001	0.001	0.003	0.001	0.001	0.002	0.001	0	0.001	0	0	0.001	0.012
	0.45 / 0.36	0.003	0.004	0.006	0.010	0.010	0.003	0.001	0.001	0	0	0.001	0	0	0.041
	0.36 / 0.28	0.004	0.009	0.019	0.018	0.009	0.003	0.002	0.001	0	0.001	0	0	0	0.069
	0.28 / 0.21	0.012	0.027	0.037	0.022	0.013	0.001	0.002	0.001	0.001	0	0	0	0	0.118
	0.21 / 0.15	0.030	0.038	0.027	0.015	0.003	0.001	0.001	0	0.001	0.001	0	0	0	0.119
	0.15 / 0.1	0.082	0.066	0.022	0.004	0.002	0.003	0.001	0	0	0	0	0	0	0.183
	0.1 / 0.06	0.135	0.058	0.010	0.002	0	0	0	0	0	0	0	0	0	0.205
	0.06 / 0.03	0.196	0.033	0.005	0	0.001	0	0	0	0	0	0	0	0	0.235
	0.03 / 0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0.000
	0.01 / 0.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0.000
Marginal Prob		0.464	0.238	0.130	0.076	0.042	0.018	0.012	0.005	0.002	0.005	0.001	0.001	0.005	1.000

Our initial experiments on the statistical distribution of loads in realistic power grids show that, like generation capacities, about 99% of the loads follow an exponential distribution with about 1% having extremely large demands falling out of the normal range defined by the expected exponential distribution. Fig. 5 and 6 show the statistical distribution of loads in the WECC-16994 and NYISO-2935 bus system. The fitting curve is depicted as a dashed line for the distribution function of P_L. The straight line in the log plot implies that about 99% of loads in the WECC system tend to follow an exponential distribution function with a mean value of 42.51 MW.

Given a realistic power grid with N buses among which N_L buses have loads, we may examine the correlation between the total number of branches connecting a bus (that is, its node degree) and the total load attached to the bus, as we did in section III for generation capacities. Following the statistical analysis as before, we consider the normalized node degree presented in (9) normalized for load as:

$$\overline{P_{L_n}} = P_{L_n} / \max_i P_{L_i} \qquad (13)$$

Note that the normalized loads will assume values in the interval [0, 1]. Our statistical results show that for realistic power grids the Pearson's coefficient of correlation varies in the range of 0.3 - 0.6. Fig. 7 displays the scatter plot of normalized loads and normalized node degree for the WECC-16994 bus system which can be further used to generate the

2-D empirical PDF for sample grids like WECC-16994 buses system.

As before, by averaging the statistics of available realistic grid data, we may extract an empirical 2-dimensional probabilistic density function (PDF) for the normalized load values and nodal degree $\left(\overline{P_{L_n}}, \overline{k_n}\right)$. Based on the 2-D empirical PDF over the obtained uneven grid division (see Fig. 8) a two-dimensional probability distribution table shown in Table III can be formulated to enable an algorithm to assign the generated load values to each load bus in a grid according to its normalized nodal degree.

The approach to creating a statistically correct random set of loads and assigning them to the load buses begins by determining the aggregate load $P_L^{tot}(N)$ using equation (11) and generating a statistically set of N_L loads which follows an exponential distribution of generation capacities with 1% of generated loads switched to super large values. In order to accurately formulate a synthetic power grid we need to assign generated loads to the load buses in a way consistent with that of a realistic grid. Therefore, the proposed approach will be developed to assign the generated loads to each load bus with respect to the statistical pattern presented in Table II. Expect for the statistical pattern, the procedure is exactly like that of generation capacities assignment in section III. The flowchart of the proposed algorithm is depicted in Fig. 9.

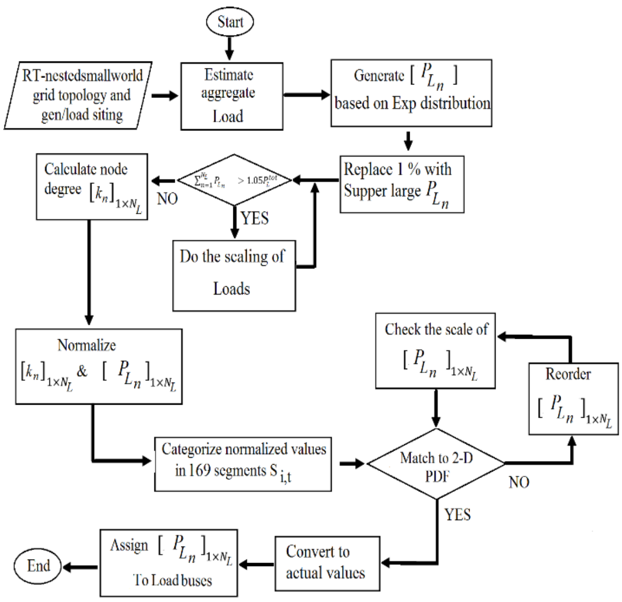


Fig. 9. Algorithm flowchart to assign random loads to load buses

V. *Conclusion and future work*

This paper presents our results on the statistics of generation and load settings for synthetic grid modeling. In this paper we examine the statistical features of generation capacities and loads in realistic power grids in terms of aggregate generation capacity, distribution of individual capacities, and their correlation with nodal degrees. Our study on the statistical distribution of both variables shows that more than 99% of the generation units/loads follow an exponential distribution with about 1% having extremely large capacities falling out of the normal range defined by the expected exponential distribution. Based on the obtained results presented in this paper there exists a non-trivial correlation between the total number of branches connected to a bus and the total generation/load attached to the bus.

Based on the above results, we develop algorithms to generate a statistically correct random set of generation capacities and loads, and then assign them to each generation and load bus. In the future work these results will be used to determine the generation dispatch at each generation bus according to its generation capacity and the statistic of dispatch ratios, and to develop a statistically based algorithm to solve the transmission line capacity assignment problem in synthetic power grid modeling.

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